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UNITED STATES AIR FORCE

APPLICATION OF THE KALMAN FILTER
TO ORBIT DETERMINATION

THESIS

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GGC/EE/70-7

SCHOOL OF ENGINEERING

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167

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APPLICATION OF THE KALMAN FILTER
TO ORBIT DETERMINATION

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
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in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

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Graduate Guidance and Control

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Preface

This thesis, a continuation and expansion of a topic first developed in a term problem by Lt. Joseph Orwat, AFITSE, GGC-69, deals with the use of the Kalman Filter to determine the orbits of near-earth satellites. The study is restricted to non-thrusting vehicles, and actual radar observations are used to maintain as much realism as possible.

We wish to express our appreciation to Lt. Col. Roger W. Johnson, AFITSE, our sponsor and advisor, who suggested the topic and provided invaluable assistance. We also wish to thank Capt. Rooch and Capt. Murphy, orbital analysts for the Space Detection and Tracking System, for their help in supplying radar data and for answering questions relating to current orbit determination methods. We are further indebted to Lt. Col. R.A. Hannen, AFITSE, for his advice and instruction concerning estimation theory. Finally, we wish to express our appreciation to our wives for their help and understanding throughout the months of work involved in this thesis.

Contents

Contents

List of Figures

List of Figures

List of Figures

List of Figures

List of Figures

List of Figures

List of Tables

List of Symbols

$\underline{X}$	Total state of the system
$\underline{x}$	Linear system state
F	System description matrix
M	Measurement matrix
$\underline{z}$	Measurement vector
$\underline{v}$	Random measurement noise vector
K	Kalman gain matrix
P	Error covariance matrix
Φ	State transistion matrix
t	Time
I	Square identity matrix
R	Measurement error covariance matrix
f_1, f_2, f_3	Equations of motion of vehicle
μ	Mass times gravitational constant
ω	Z component of the earth's rotation rate
Δ	Variation or incremental change
x, y, z	Geocentric coordinates of satellite
X, Y, Z	Geocentric coordinates of tracking stations
r	Position vector of satellite
J_2, J_3	Zonal harmonic coefficients
ρ	Slant range
ϕ	Latitude of tracking station
θ	Longitude of tracking station
h	Height of tracking station

A	Azimuth angle of vehicle
E	Elevation angle of vehicle
a	Semi-major axis
e	Eccentricity
i	Inclination angle of vehicle
(-)	Immediately prior to a measurement
(+)	Immediately following a measurement

<u>Subscripts</u>

I	References inertial geocentric system
E	References rotating geocentric system
T	References rotating topodetic system
m	Measured value
n	Nominal value
a	Actual value
$\underline{(\ )}$	Vector quantity
o	Initial value

<u>Superscripts</u>

*	Nominal or reference value
$(\dot{\ })$	First derivative with respect to time
$(\ddot{\ })$	Second derivative with respect to time
^	Best estimate
T	Transpose
-1	Inverse

Abstract

As more satellites are launched into space, a great need exists for a rapid, recursive (reducing computer memory requirements) orbit determination method. This paper presents such a method by applying Kalman Filter theory to determine the position and velocity of near-earth satellites using data from a fixed observer on the Earth.

The vehicle equations of motion are linearized in a Taylor Series expansion. The nominal states, position and velocity, are determined by integrating the nonlinear equations of motion, and the linear filter theory is used to estimate the errors in these states. The linear estimated errors are added to the nominal states to obtain an updated trajectory which is used as the starting point on a new nominal for the next integration.

Actual tracking data from four different satellites are used in the study. Convergence of the error estimates to values less than 0.1 per cent of the best estimates of position and velocity is obtained within 50-250 seconds from the time of the initial radar contact. The program is capable of integrating for over 80 seconds with no tendency to diverge. Several orbital elements are computed, and these compare quite closely with results supplied by the Space Detection and Tracking System. The rate of convergence is related to the initial guess

of the error covariance matrix, along with the measurement accuracy of the tracking stations.

The paper includes the Kalman Filter equations along with the iterative procedure needed to process the filter. This is followed by an explanation of the coordinate systems, the formulation of the dynamic equations of motion, and the computer algorithm used. Results are presented in both tabular and graphical form.

APPLICATION OF THE KALMAN FILTER TO ORBIT DETERMINATION

I. Introduction

The job of quickly and accurately determining the orbit of near-earth satellites is rapidly becoming a major problem as more and more objects are launched into space. One of the agencies assigned such a task is the Space Detection and Tracking System (SPADATS), located at Ent Air Force Base in Colorado Springs, Colorado. Each day they are faced with the task of locating, tracking, and cataloging several thousand objects orbiting the Earth.

In order to determine the orbital path of a vehicle, it is first necessary to accurately determine both the vehicle's position and velocity. However, since these quantities are not directly observable, it is necessary to infer them from a sequence of observations obtained from radar or photographic tracking devices. Once the position and velocity are known, they can be used to calculate the desired orbital elements. Normally six elements are required to precisely describe the orbit dimension, the orientation of the vehicle's path with respect to the Earth, and to locate the vehicle along that path at any given time.

In conventional orbit determination methods, it is first necessary to obtain a preliminary orbit. Sometimes

this can be determined directly from the nominal conditions computed prior to launch, but often it must be found from approximate procedures based upon actual radar observations made after launch. Several current techniques include the Lagrange-Gauss-Gibbs First Approximation, the Laplacian First Approximation, and the Herrick-Gibbs method. These methods along with several others are discussed by Baker in Ref 3:23-77.

Once the preliminary orbit is found, it is then continually updated or corrected by using differential correction techniques. The first step in this correction procedure involves the "representation" or calculation of the available observations using the adopted orbital elements. The representation may be carried out by either numerical integration (special perturbations) or analytical integration (general perturbations). The representation ends with the calculation of residuals; i.e., differences between the actual observations and those computed with the adopted elements. The differential correction theory then relates these residuals to improvements to the initial orbital elements by linear differential relationships. The residuals also give an indication of the accuracy of the elements and aid in deciding whether or not another correction is necessary (Ref 9:125). The overall purpose of the differential correction process is to define the orbital parameters so that the residuals

are minimized. Such a minimization is usually taken in the least-squares sense in that the parameters are adjusted until the sum of the squares of the residuals is a minimum (Ref 3:79-80).

Since the equations of motion are highly nonlinear, the region of linearity required for the differential correction method becomes more and more constrictive about the nominal trajectory the longer the time period over which the prediction is made. Thus, the differential correction technique, by fitting data over a long time arc, often produces a result which is outside the linear range. This produces problems in convergence and consumes machine time (Ref 5:4-5).

Since the Kalman Filter is an optimal filter applicable to noisy, time-varying, linear systems, it is particularly suited for orbit determination problems in which estimates of the state variables are desired as rapidly as possible. At each data computation cycle the filter requires only the present state of the system, the present measurement, and the associated covariance matrices. The present estimate of the state variable deviation is related to the present actual deviations in the observations. This permits a complete optimal estimate of the orbit variables and the observation errors from each single observation. As a result, computational efficiency is achieved by requiring only

the data points that have been computed at the last cycle.

The differential correction method utilizes a large number of independent data in an after-the-fact manner to improve orbital parameters. However, the Kalman Filter method not only provides a recursive procedure to process each observation as it occurs, but also yields a predicted set of orbital parameters coupled with a matrix of coefficients which depicts the accumulated error in the predictions in real time. This recursive procedure may be used for the following: (1) satellite or ICBM interception, (2) to obtain knowledge of current errors in prediction. It should also be pointed out that the Kalman Filter method is equally applicable to orbit determination for either an orbital observer or a fixed observer with only slight modification (Ref 15:67-75).

Statement of the Problem

The problem to be considered involves the orbit determination of a space vehicle by the application of the Kalman Filter. The required input data consists of a series of radar observations of the vehicle from a tracking station on the earth, along with the actual location of the station. These radar observations include range, range rate, azimuth, elevation, azimuth rate, and elevation rate. The main objective of the problem is to determine the position and velocity of

the space vehicle by using Kalman Filter techniques and the data provided from the observation point on the earth. Then by using these results several orbital elements can be calculated such as the semi-major axis, eccentricity, and inclination angle.

Assumptions and Limitations

The equations of motion of the vehicle are derived using the assumption that the vehicle has negligible mass and is under the gravitational action of a single dominant central force field, the earth. The perturbations due to the earth's oblateness are included in the program, but other external perturbations and system disturbances, such as atmospheric and solar drag, are assumed negligible. Also, only the motion of non-thrusting satellites are considered, and any uncertainties in the latitude, longitude, or height of the tracking stations are considered negligible.

Plan of Development

This report is divided into six chapters plus an Appendix. Chapter II presents the Kalman Filter equations and describes the basic iterative procedure needed to implement these equations. This is followed in Chapter III by an explanation of the coordinate systems used, the formulation of the dynamic equations of motion for an orbiting space vehicle, and the linearization

procedure. Chapter IV describes the basic orbit determination algorithm along with the computer program used to implement it. Chapter V presents and analyzes the results obtained, and conclusions and recommendations are given in Chapter VI. The initial nominal trajectory equations, the derivation of the system description matrix, the initial trajectory equations, a listing of the computer program, and additional graphical results are included in the Appendixes.

II. The Kalman Filter Equations

Linear System

The Kalman Filter is a recursive data processing technique. It utilizes the measurement covariance matrix, the state covariance matrix, a dynamic vehicle model, and the model noise statistics, to provide a minimum-variance estimate of the state variables in a nonstationary linear system. The basic Kalman Filter equations are presented in this chapter, while an actual derivation can be found in Ref 6:125-128.

The covariance matrix for two random processes is defined in terms of the ensemble average values of the vectors and the ensemble average value of their outer product. The covariance matrix for a(t) and b(t) is given by (E denotes ensemble expectation)

$$\mathrm{cov}\left[a(t),b(t)\right] = E\left[\underline{a}(t)\underline{b}^T(t)\right] - E\left[\underline{a}(t)\right]E\left[\underline{b}^T(t)\right] \tag{1}$$

Since only zero mean quantities are dealt with in Kalman Filter work, the simplification

$$\mathrm{cov}[\underline{a}(t),\underline{b}(t)] = E[\underline{a}(t)\underline{b}^T(t)] \tag{2}$$

can be made. The covariance of the errors in the Kalman Filter estimate of the state $\underline{x}$ is described by the matrix P

$$P(t) = \mathrm{cov}\left[\underline{\tilde{x}}(t),\underline{\tilde{x}}(t)\right] \tag{3}$$

For measurement noises, the covariance is given by

$$R = \text{cov}(v,v) \tag{4}$$

When the discrete version of the Kalman Filter is used, the requirement that the measurement noise and system disturbances be uncorrelated over the smallest measurement interval gives the restrictions

$$\text{cov}(\underline{v}_m,\underline{v}_n) = 0 \qquad \text{for } m \neq n \tag{5}$$

$$\text{cov}(\underline{v}_m,\underline{v}_n) = R_n \qquad \text{for } m = n \tag{6}$$

Thus the measurement covariance matrix relates the state variables to the quantities being measured, while the state covariance matrix describes the uncertainty in the optim[illegible] estimate of the state vector. The diagonal terms of the state covariance matrix are the variances in the estimation error for each of the individual state variables. The off-diagonal terms are a measure of the cross-correlation between the state variables.

The state variables of a linear system are written in the form of a state vector

$$\underline{x} = \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \tag{7}$$

The vector notation permits the linear system to be described by the unforced matrix differential equation

$$\dot{\underline{x}} = F\underline{x} \tag{8}$$

where F is the system description matrix. The measurements in the Kalman Filter are taken as linear combinations of the system state variables, corrupted by uncorrelated noise. The measurement equation is written as

$$\underline{z} = M\underline{x} + \underline{v} \tag{9}$$

where $\underline{z}$ is the set of measurements arranged in vector form, M is the measurement matrix which describes the linear combination of state variables which comprise $\underline{z}$ in the absence of noise, and $\underline{v}$ is a vector of random noise quantities corrupting the measurements.

One way to use the measurement vector $\underline{z}$ is to anticipate it based on knowledge of the measurement matrix M_n and the estimate of the state vector at the instant the measurements are taken. If $\underline{z}_n$ and $M_n\underline{x}_n$ do not agree, the difference must result from the measurement noise $\underline{v}_n$ or an error in the estimate. The state variable estimates can then be changed according to statistical knowledge of the errors in $\underline{x}_n$ and of the measurement errors. Thus, the state error vector estimate after consideration of the measurement is given by

$$\hat{\underline{x}}_n(+) = \hat{\underline{x}}_n(-) + K_n\left[\underline{z}_n - M_n\hat{\underline{x}}_n(-)\right] \tag{10}$$

where the (-) and (+) indicate immediately prior to and after a measurement respectively. The filter gain matrix is determined by

$$K_n = P_n(-)M_n^T\left[M_nP_n(-)M_n^T + R\right]^{-1} \tag{11}$$

Between measurements, the state error vector estimate is determined from

$$\hat{\underline{x}}_{n+1} = \Phi_n\hat{\underline{x}}_n \tag{12}$$

where Φ, the state transition matrix, is computed from the system dynamics as expressed in the system matrix F. When F is a constant, Φ is a function of the time difference Δt, and is represented by the series

$$\Phi = I + F\Delta t + F^2\Delta t^2/2! + \cdots \tag{13}$$

The use of measurements provided at discrete instants of time causes the error covariance to be discontinuous, having different values before and after the measurements. The value prior to a new measurement is determined by

$$P_{n+1}(-) = \Phi_nP_n(+)\Phi_n^T \tag{14}$$

where Φ is the state transition matrix defined in Eq (13).

If Eq (10) is used to improve the state vector estimate, the new error covariance is expressed by

$$P_n(+) = (I-K_nM_n)P_n(-)(I-K_nM_n)^T + K_nR_nK_n^T \tag{15}$$

where I is the square identity matrix and R_n is the covariance matrix of the measurement errors $\underline{v}_n$. This equation can be written in other forms, but Eq (15), due to its symmetry, is preferred since it is less sensitive to computational inaccuracy (Ref 11:261).

Extension to Non-Linear Systems

Since the Kalman Filter is an optimal filter applicable to noisy, time-varying, linear systems, it is necessary to linearize the vehicle equations of motion in a Taylor series expansion about a nominal trajectory. The nominal states, position and velocity, are determined by integrating the nonlinear equations of motion, and the Kalman Filter theory is then used to estimate the errors in the nominal states. The linear filter theory is applied to the estimates of the errors in the states since these errors in a nonlinear system behave much more linearly than the states themselves. By adding these linear estimated errors to the nominal states, an updated trajectory is obtained for use as the starting point on a new nominal for the next integration. This nominal estimated trajectory is therefore being used as the reference about which the linearization is done. This can cause large estimated errors initially since the initial estimated trajectory may be far from the actual trajectory, thereby resulting in a violation of the basic linearity assumptions. As more measurements are processed,

however, the recursive action of the Kalman Filter will cause the updated nominal to approach the actual trajectory.

Many past applications of Kalman Filtering to non-linear systems involved linearization about a pre-computed nominal trajectory. This approach allows for the pre-computation of the error covariance matrices P(-) and P(+). An application of Kalman Filtering to a near-nominal ascent guidance problem is given by J. A. Henz in Ref 13. The problem associated with a pre-computed nominal is that noise either in the measurements, the random system disturbances, or both, could cause the linearized equations to be totally unsatisfactory approximations to the true nonlinear equations.

This problem then becomes the motivation for the approach of continuously expanding about the latest optimal estimate of the nominal trajectory. The reasoning is that if the filter is working fairly well, the updated nominal value should be much closer to the actual trajectory than the old nominal value. As a result, the first order Taylor series expansions about the new nominal will naturally be more accurate approximations. The success of the filter which results from linearizing about the latest nominal trajectory as it becomes available has, in fact, been so good that it has become the "work-horse" recursive filter for almost all real applications (Ref 12:35).

Data Needed For Kalman Filter

In order to apply the Kalman Filter, certain information about the system and the statistical characteristics of the input and measurement noises must be known or assumed. The following data is required to initialize the Kalman Filter process:

1. System description matrix F
2. Sampling time Δt
3. State transition matrix Φ
4. Measurement matrix M
5. Measurement noise covariance matrix R
6. Initial state covariance matrix P(0)
7. Initial state estimate $\hat{x}(0)$

The block diagram for the discrete Kalman Filter is shown in Fig. 1.

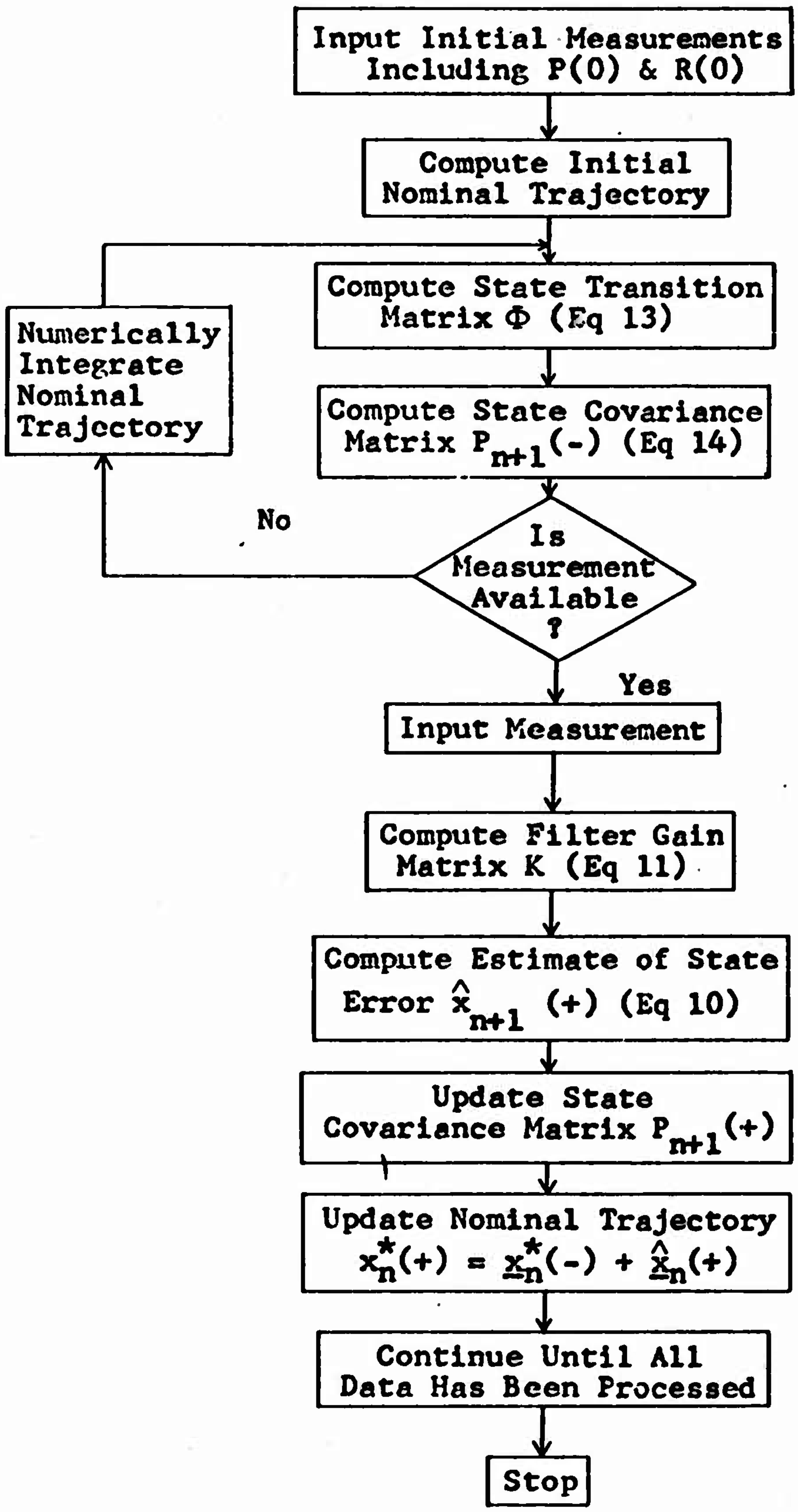

Fig. 1 Flow Chart For Discrete Kalman Filter

III. Equations of Motion

Coordinate Reference Systems

The position of a satellite may be referred to a wide variety of coordinate systems. These systems differ primarily in the choice of origin and fundamental reference plane. Two coordinate origins are involved in the satellite problem: the radar observer (rotating topodetic system), and the center of the Earth (geocentric system) (Ref 9:7). The geocentric system is further divided by referring to an inertial geocentric system and a rotating geocentric system. The three coordinate systems are shown in Fig. 2.

Inertial Geocentric System. The center of the earth is the origin of the inertial geocentric system, and the principal direction, X_I, is toward the mean vernal equinox of date. The positive Z_I-axis is in the direction of the mean north celestial pole and the Y_I-axis completes a right-handed orthogonal coordinate system. This is the coordinate frame in which the orbital elements are computed.

Rotating Geocentric System. The center of the earth is the origin of the rotating geocentric system, and the principal direction, X_E, is toward the intersection of the prime meridian, Greenwich, and the equator. The positive Z_E-axis is in the direction of the north

celestial pole and the Y_E-axis completes a right-handed orthogonal system. This coordinate system serves as the reference for the equations of motion of the satellite.

Rotating Topodetic System. The third coordinate system, the rotating topodetic, has an origin at the tracking station (topos). The principal direction, X_T, is toward the local North direction on the target plane to the spheroid model. The positive Z_T-axis is normal to the spheroid model, and the Y_T-axis completes a right-handed system. The earth model used as the reference spheroid is described in Ref 1. Since this rotating topodetic system is used as the reference frame for the observed values of the measurements, it is convenient to also use it for the calculated values.

Initial Nominal Trajectory

The initial nominal trajectory (position and velocity) is determined from the first set of input data provided by the radar tracking station. This data includes the range, range rate, azimuth, elevation, and azimuth and elevation rate of the satellite being tracked. The equations used to compute the position and velocity of the satellite in the rotating geocentric system are given below, and the individual vector components are presented in Appendix B.

$$\underline{r} = \rho \underline{L} - \underline{R} \tag{16}$$

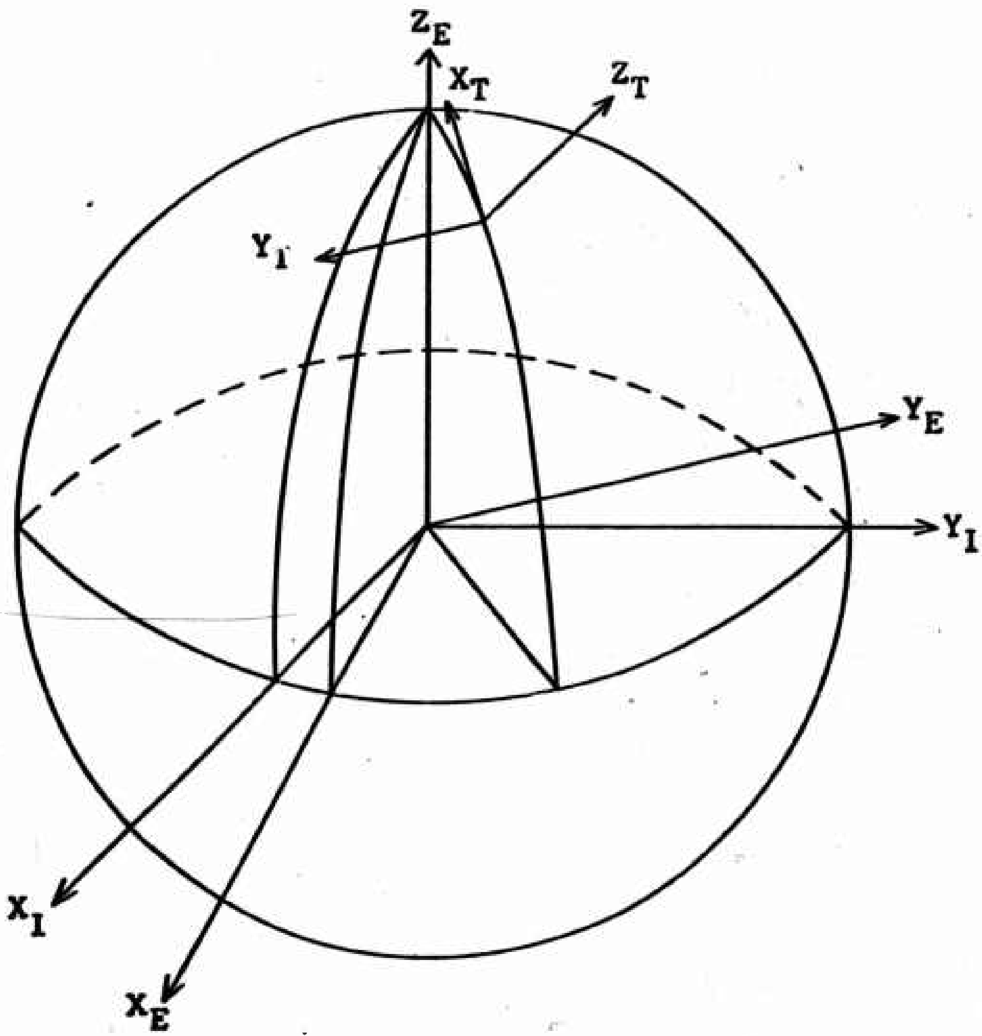

Fig. 2. Coordinate Systems

$$\dot{\underline{r}} = \dot{\rho}\underline{L} - \rho\dot{\underline{L}} \tag{17}$$

Equations of Motion

The nonlinear equations for the vehicle in the rotating geocentric coordinate system are given by

$$\ddot{x} = f_1 = -\frac{\mu x}{r^3}(1+\text{Harmonic A}) + 2\omega\dot{y} + x\omega^2 \tag{18}$$

$$\ddot{y} = f_2 = -\frac{\mu y}{r^3}(1+\text{Harmonic A}) - 2\omega\dot{x} + y\omega^2 \tag{19}$$

$$\ddot{z} = f_3 = -\frac{\mu z}{r^3}(1+\text{Harmonic B}) \tag{20}$$

Centripetal force and Coriolis force terms are included in the equations since the accelerations are referenced to a rotating frame. The harmonic terms are given by

$$\text{Harmonic A} = 1.5J_2(A_e/r)^2\left[1-5(z/r)^2\right] + 2.5J_3(A_e/r)^3\left[3-7(z/r)^2\right]z/r \tag{21}$$

$$\text{Harmonic B} = 1.5J_2(A_e/r)^2\left[3-5(z/r)^2\right] + 2.5J_3(A_e/r)^3\left[6-7(z/r)^3\right]z/r \tag{22}$$

where J_2 and J_3 are zonal harmonic constants which represent the deviation of the earth from a perfect sphere. All subsequent zonal harmonic terms and all of the tesseral harmonic terms are neglected in order to simplify the Taylor Series expansion of the nonlinear equations of motion. A complete discussion of the equations of motion

and the harmonic terms can be found in Baker (Refs 2:144-145, 3:167-183). Equations (18), (19), and (20) actually represent a set of six first order differential equations or a 6 x 1 state vector differential equation, where the first three states represent position and the latter three velocity, all in Cartesian coordinates. Next, the non-linear equations of motion are expanded in a Taylor Series about a nominal trajectory:

$$\ddot{x} = f_1(x^*,y^*,z^*,\dot{x}^*,\dot{y}^*,\dot{z}^*) + \frac{\partial f_1}{\partial x}(x-x^*) + \frac{\partial f_1}{\partial y}(y-y^*) + \frac{\partial f_1}{\partial z}(z-z^*) + \frac{\partial f_1}{\partial \dot{x}}(\dot{x}-\dot{x}^*) + \frac{\partial f_1}{\partial \dot{y}}(\dot{y}-\dot{y}^*) + \frac{\partial f_1}{\partial \dot{z}}(\dot{z}-\dot{z}^*) + \text{h.o.t.} \tag{23}$$

with similar equations for y and z (24)-(25)

Now a linear state vector or error estimate vector $\underline{x}$ is defined as the difference between the actual and nominal state vector.

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} X - X^* \\ Y - Y^* \\ Z - Z^* \\ \dot{X} - \dot{X}^* \\ \dot{Y} - \dot{Y}^* \\ \dot{Z} - \dot{Z}^* \end{bmatrix} \tag{26}$$

Combining Eqs (23), (24), and (25) with Eq (26), and

retaining only first order terms, results in the linear state vector differential equation

$$\dot{\hat{\underline{x}}} = F\hat{\underline{x}} \tag{27}$$

where F is defined as

$$F = \left[\begin{array}{c|c} 0 & I \\ \hline \multicolumn{2}{c}{F(t)} \end{array}\right] \tag{28}$$

and F(t) is given by

$$F(t) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \cdots & \frac{\partial f_1}{\partial \dot{z}} \\ \vdots & & & \vdots \\ \frac{\partial f_3}{\partial x} & \cdots & \cdots & \frac{\partial f_3}{\partial \dot{z}} \end{bmatrix} \tag{29}$$

All the partial derivatives in Eq (29) are evaluated along the nominal trajectory. The actual derivation of the system description matrix, F(t), is given in Appendix B. Application of the Kalman Filter provides the best estimate of the linear state vector, $\hat{\underline{x}}$, which is then used in Eq (10) to obtain the latest best estimate of the nominal trajectory.

Coordinate Transformation

The measurement data includes slant range, slant range rate, azimuth, and elevation as shown in Fig. 3.

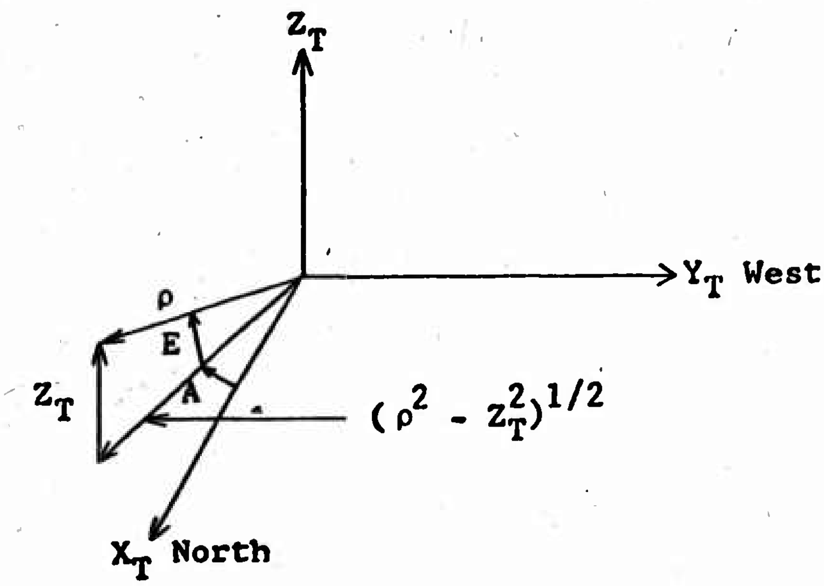

Fig. 3. Measurement Geometry

Because the calculated coordinate values of the slant range vector are desired in the rotating topodetic coordinate system, a transformation is needed to go from the rotating geocentric system to the rotating topodetic system. Since the slant range vector is the sum of the geocentric position vector of the tracking station and the geocentric position vector of the satellite (Fig. 2), then the coordinate transformation is given by

$$\begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} = \begin{bmatrix} -\sin(\phi)\cos(\theta) & -\sin(\phi)\sin(\theta) & \cos(\phi) \\ \sin(\theta) & -\cos(\theta) & 0 \\ \cos(\phi)\cos(\theta) & \cos(\phi)\sin(\theta) & \sin(\phi) \end{bmatrix} \begin{bmatrix} x+X \\ y+Y \\ z+Z \end{bmatrix} \quad (30)$$

This is an orthogonal transformation matrix, where ϕ and θ represent the latitude and longitude of the tracking station respectively, and X_T, Y_T, and Z_T are the topodetic coordinates of the range vector.

Measurement Vector Expansion About The Nominal Trajectory

The slant range, slant range rate, azimuth, and elevation are calculated using Eqs (31)-(34)

$$\rho = \left[(x+X)^2 + (y+Y)^2 + (z+Z)^2\right]^{1/2} \tag{31}$$

$$\dot{\rho} = 1/\rho\left[(x+X)^2\,\dot{x} + (y+Y)^2\,\dot{y} + (z+Z)^2\,\dot{z}\right] \tag{32}$$

$$A = \mathrm{TAN}^{-1}\left[-Y_T/X_T\right] \tag{33}$$

$$E = \mathrm{TAN}^{-1}\left[Z_T/(\rho^2 - Z_T^{\,2})^{1/2}\right] \tag{34}$$

These equations are based on the geometry of Figs. 2-3. Equation (32) represents only the radial component of the slant range rate vector, and is derived by taking the dot product of the slant range rate and slant range vectors.

The measurements may be expressed in the form

$$\begin{aligned} \rho_m &= \rho_a + v_1 \\ \rho_m &= \rho_a + v_2 \\ A_m &= A_a + v_3 \\ E_m &= E_a + v_4 \end{aligned} \tag{35}$$

where the subscripts "a" and "m" denote the actual values and the measured values respectively, and v_i represents zero mean, Gaussian-distributed white noise.

Next, define

$$\begin{aligned} \Delta\rho_m &= \rho_m - \rho_n \\ \Delta\dot{\rho}_m &= \dot{\rho}_m - \dot{\rho}_n \\ \Delta A_m &= A_m - A_n \\ \Delta E_m &= E_m - E_n \end{aligned} \tag{36}$$

where the subscripts "n" and "m" denote the nominal values and the measured values respectively. Now let

$$\begin{aligned} \Delta\rho &= \rho_a - \rho_n \\ \Delta\dot{\rho} &= \dot{\rho}_a - \dot{\rho}_n \\ \Delta A &= A_a - A_n \\ \Delta E &= E_a - E_n \end{aligned} \tag{37}$$

This allows Eq (35) to be rewritten as

$$\begin{bmatrix} \Delta\rho_m \\ \Delta\dot{\rho}_m \\ \Delta A_m \\ \Delta E_m \end{bmatrix} = \begin{bmatrix} \Delta\rho \\ \Delta\dot{\rho} \\ \Delta A \\ \Delta E \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \tag{38}$$

The quantities $\Delta\rho$, $\Delta\dot{\rho}$, ΔA, and ΔE represent deviations from from the nominal, and if these are assumed small, then a first order Taylor Series expansion about the nominal results in

$$\begin{bmatrix} \rho \\ \dot{\rho} \\ A \\ E \end{bmatrix} = \begin{bmatrix} \frac{\partial \rho}{\partial x} & \frac{\partial \rho}{\partial y} & \cdots & \frac{\partial \rho}{\partial \dot{z}} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \frac{\partial E}{\partial x} & \cdots & & \frac{\partial E}{\partial \dot{z}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_6 \end{bmatrix} \tag{39}$$

The linear measurement vector, $\underline{z}$, can now be defined as

$$\underline{z}(t) = M(t)\underline{x} + \underline{v}(t) \tag{40}$$

where $\underline{v}(t)$ is the noise vector and $\underline{x}$ is the linear state vector defined by Eq (26). The measurement matrix M is the matrix of partial derivatives in Eq (39), and the actual partials are derived in Appendix C. The components of M are always evaluated at the latest value of the nominal trajectory. The validity of the linear model as expressed by Eqs (27) and (40) is dependent upon the nearness of the nominal trajectory to the actual trajectory. If the estimation error is too large, then the higher-order terms neglected in the Taylor Series become important. Experience has shown that, in most cases, the assumption of a linear model is adequate for near-earth orbits, even for position errors approaching two to five miles (Ref 7:13).

Suboptimal Kalman Filter

The linear state vector differential equation given in Eq (27) is never solved in the basic algorithm. Its

derivation is necessary to produce a system description matrix, F, which is used to compute a value of the state transition matrix. Likewise, the linear measurement vector equation given in Eq (40) simply defines the measurement matrix M, which is needed for the Kalman Filter equations.

In view of the definition of the linear state variables $\underline{x}$ and $\underline{z}$, Eq (10) is presented again,

$$\underline{\hat{x}}_n(+) = \underline{\hat{x}}_n(-) + K_n\left[\underline{z}_n - M_n\underline{\hat{x}}_n(-)\right] \tag{41}$$

Let $\underline{\hat{x}}_n(-)$ be the nominal values of position and velocity obtained by integration of the nonlinear equations of motion. These values are considered to be the best estimates available just prior to a measurement,

$$\underline{\hat{x}}\,(-) = \underline{x}^* \tag{42}$$

The quantity within the brackets of Eq (41) actually represents the definition of the linear measurement vector, because $M_n\underline{\hat{x}}_n(-)$ may be computed from Eqs (31)-(34), and by letting

$$\underline{z}_{nom} = M_n\underline{\hat{x}}(-) \tag{43}$$

then

$$\underline{z} = \underline{z}_n - \underline{z}_{nom} \tag{44}$$

Equation (41) can then be rewritten as

$$\underline{\hat{x}}(+) = \underline{x}^* + K\underline{z} \tag{45}$$

By letting $\hat{\underline{x}}(+)$ be the best estimate of position and velocity after a measurement, then

$$\hat{\underline{x}}(+) = x \tag{46}$$

and

$$\underline{x} = x - x^* \tag{47}$$

Thus, Eq (45) may be expressed in terms of the linear error vector and the linear measurement vector,

$$\hat{\underline{x}}_n = K_n \underline{z}_n \tag{48}$$

It must be emphasized that the filter which is represented in Eq (41) is not the exact optimal filter for the nonlinear system of Eqs (8) and (9). Rather, it is a first-order approximation which is valid only to the extent that the linearized model in Eqs (27) and (40) is a valid approximation of the original nonlinear system. Basically, what has been done here is to use the linear theory as a guide to obtain an approximate solution of a nonlinear filtering problem (Ref 8:11-15). A Kalman Filter used in this manner is frequently referred to as a suboptimal Kalman Filter.

IV. Program Description

This section gives a brief description of the computer program written to apply the Kalman Filter to the problem of orbit determination. The program is run on the IBM 7094 computer located at Wright Patterson Air Force Base, Ohio. A complete listing of the program, a listing of all the variables used in the program, and an example of the input data required, is found in Appendix C.

Computer Algorithm

The following is the basic algorithm used to implement the Kalman Filter in an orbit determination problem.

1. Input ρ, $\dot{\rho}$, A, E, $\dot{A}$, $\dot{E}$, Ht, ϕ, θ, P(0), R(0)
2. Compute initial nominal trajectory $\underline{x}^*$($\underline{r}$ and $\dot{\underline{r}}$)
3. Compute orbital elements
4. Calculate estimates of state errors: $\hat{\underline{x}} = K\underline{z}$
5. Update nominal trajectory $\underline{x}_n^*(+) = \underline{x}_n^*(-) + \hat{\underline{x}}_n(+)$
6. Update error covariance matrix from t_n to t_{n+1}
7. Numerically integrate non-linear equations of motion from t_n to t_{n+1}
8. Check to see if new measurement data is available:
   a. Yes - Go to step 4
   b. No - Go to step 6
9. Continue until all data has been processed.
10. Recompute orbital elements.

Description of Computer Program

The computer program consists of a MAIN program plus additional subroutines needed to implement the various steps of the basic algorithm.

MAIN Program. The MAIN program calls the operational subroutines in the following sequence:

1. Call ZERO to initialize the various constants needed in the program.
2. Call INPUT to input range, range rate, station height, latitude and longitude of station, initial state error covariance matrix, and measurement error covariance matrix.
3. Call STAT to compute the coordinate transformation matrix.
4. Call EQCOMP to calculate initial nominal trajectory ($\underline{r}$ and $\dot{\underline{r}}$).
5. Call ELEMTS to compute orbital elements.
6. Update the nominal trajectory by calling KALMAN.
7. Call UPCOV to update the error covariance matrix from t_n to t_{n+1}.
8. Numerically integrate nonlinear equations of motion (supplied by DERT) in TRAJ.
9. Check for new measurement data:
   a. If available, return to step 6.
   b. If not available, return to step 7.
10. Continue the above procedure until all measurement

data is processed.

11. Call DRAW to plot results.

Subroutines. A brief description of each of the subroutines used in the program is given below:

ZERO - Initializes the various constants needed in the program

INPUT - Reads in the following initial data:

Card 1 - Integration step size

Card 2 - Station biases

Card 3 - Initial error covariance matrix

Card 4 - Initial measurement covariance matrix

Card 5 - Ephemeris starting time

Card 6 - Latitude, longitude, height of station

Card 7 - Initial measurements

INPUTA - Reads in the second and subsequent radar observation cards. The format is consistent with SPADATS data cards.

OUTPTA - Lists the station parameters and radar measurements (input data), satellite position and velocity, and orbital elements (output data).

UPCOV - Updates the error covariance matrix (Eq 14) from time t_n to t_{n+1}.

EQCOMP - Computes the equatorial components of

position and velocity for use as the initial nominal trajectory. The equations used are found in Ref 2:118-130, and Appendix A.

ELEMTS1 - Computes the orbital elements when given the equatorial components for position and velocity. P, Q, and W unit vectors are used and the actual equations are given in Ref 3:11-22.

ELEMTS2 - Computes the orbital elements when given the equatorial components for position and velocity. U, V, and W unit vectors are used and the actual equations are given in Ref 10:55-60.

STAT - Computes the coordinate transformation matrix from the latitude and longitude of the station.

MEAS - Computes the range, range rate, azimuth, and elevation (Eqs 31-34) for comparison with the measured values from the tracking station. The measurement matrix (Appendix C) is also computed for use in KALMAN.

DERT - Provides the vehicle equations of motion (Eqs 18-20) to be numerically integrated.

TRAJ - Numerically integrates the nonlinear equations of motion using an Adams-Moulton

integration procedure. This subroutine is a modified form of the IBM 360 subprogram DFEQ.

KALMAN - Processes the filter equations and provides the best estimate of the errors in position and velocity following a radar observation. This error estimate is then used to update the nominal trajectory.

SDM - Computes the system description matrix (Eq 29).

<u>Subordinate Subroutines</u>. These subroutines are used within the operational subroutines to perform mathematical and plotting operations.

DET - Computes the determinant of a function

SINV - Inverts a given symmetric positive definite matrix

MFSD - Factors a given symmetric positive definite matrix

MPRD - Multiplies two matrices

LOC - Computes a vector subscript for an element in a matrix of specified storage mode (needed for MPRD)

MTRA - Transposes a matrix

MCPY - Copies an entire matrix (needed for MTRA)

DRAW - Draws a graph using Cartesian coordinates

PLOTS - Initiates the plotting routine

PLOTE - Terminates the plotting routine

V. Presentation and Analysis of Results

Available Data

The data used in the computer simulation was provided by the Space Detection and Tracking System (SPADATS), located at Colorado Springs, Colorado. Four different satellites, whose orbital paths ranged from highly elliptical to nearly circular, are used in the study. The input radar measurements consist of the range, range rate, azimuth, elevation, and azimuth and elevation rates of the satellites being tracked. Since the azimuth and elevation rates are not always provided, it is sometimes necessary to estimate them based on two consecutive values of azimuth and elevation. This estimate is only required for the determination of the initial nominal trajectory. For all subsequent observations only range, range rate, azimuth, and elevation inputs are needed.

The radar stations used to obtain the necessary tracking data are listed in Table I along with their latitudes, longitudes, height above sea level, and sensor sigmas and biases. The sensor sigmas are squared and used as the diagonal terms for the measurement error covariance matrix R, and the sensor bias values are used to correct the measurement values at each observation time. The bias corrections are necessary because of the assumption of a zero mean noise vector. Table II

Table I

Radar Tracking Stations

Station Parameter	Station 342	Station 348	Station 349	Station 337	Station 345
Latitude (Deg)	+54.37	+76.57	+64.29	+37.01	+52.73
Longitude E. (Deg)	+359.33	+291.71	+210.81	+39.99	+174.10
Height (meters)	276.00	370.00	240.00	915.00	93.00
Sigmas					
Range (KM)	0.50	1.00	1.20	0.09	0.06
Azimuth (Deg)	0.02	0.06	0.03	0.10	0.02
Elevation	0.02	0.05	0.03	0.07	0.03
Range Rate (M/S)	2.00	2.00	2.00	5.00	1.00
Bias					
Range (KM)	-0.40	-0.80	-1.30	+0.30	+0.03
Azimuth (Deg)	+0.03	+0.05	+0.04	-0.03	-0.08
Elevation (Deg)	+0.02	+0.06	-0.02	-0.32	+0.18
Range Rate (M/S)	-15.00	0.00	-2.00	+1.00	0.00
Tracking Intervals (Sec)	10.2	10.1	10.1	6.0	6.0

Table II

Nominal Elements Computed Before Launch

Element	Satellite 3823	Satellite 3824	Satellite 3825	Satellite 3826
Period (Min)	93.83	95.27	154.04	90.84
Inclination (Deg)	99.42	99.05	105.00	99.28
Eccentricity	0.000008	0.011	0.28	0.02
Apogee (NM)	249.4061	330.000	3142.70	245.91
Perigee (NM)	249.3481	249.387	249.05	94.99
Argument of Perigee (Deg)	172.36	179.32	178.20	106.00
Semi-Major Axis (NM)	3693.31060	3733.627	5139.81	3614.38
Injection Height (NM)	249.3481	249.388	249.05	245.91
Injection Latitude (Deg)	7.691	7.696	1.74	2.35
Injection Longitude (Deg)	228.809	228.805	226.71	4.09
Injection Time After L/O (Sec)	699.2	731.9	970.7	3239.1

(From Ref 11:2)

presents the nominal elements for each of the satellites as computed prior to launch.

The computed orbital elements obtained in this study are compared with values supplied by SPADATS. For three satellites, only one set of elements for both days covering the time of observations are provided by SPADATS. In addition, the exact time of day when the orbital elements are calculated by SPADATS is not available. As a result, it is usually impossible to have the times of computation coincide, thereby preventing any precise comparison of results.

Introduction to Computer Results

The computer runs used to evaluate the application of the Kalman Filter to orbit determination are divided into four groups. The first group of runs are used to determine the basic effectiveness of the filter when applied to tracking data for each of the four satellites. The second group of computer runs use observations from the same pass as the initial runs, but from different stations, and also data from one to three passes earlier or later than the initial runs. These runs provide a means of checking the changes in the results caused by different tracking stations and different passes. The third group of runs investigates what would happen if several observations are skipped from within a series of observations. The last group consists of several

runs to check the effects of varying the number of terms in the equation used to compute the state transition matrix. A run is also included which utilizes the longest single track of observations found in the available data.

Group I Results

The initial computer simulation is made up of ten computer runs as shown in Table III. The input data for each run consists of a series of radar observations made by a single tracking station during one pass of the satellite. The satellite is usually tracked over a period of four to five minutes during each pass, and observations are recorded at intervals of either 10.1 or 6.0 seconds. In order to begin the filter algorithm, it is necessary to estimate the initial error covariance matrix P(0). At the start of each run, the linear error vector is expected to be zero since the satellite is assumed to be on the nominal trajectory. The confidence in this assumption is expressed by the initial values of the covariance matrix. The recommended procedure is to underestimate the off-diagonal (cross-correlation) terms and overestimate the diagonal (auto-correlation) terms. Therefore, all the off-diagonal terms of the 6 x 6 error covariance matrix are initially set to zero, and the different combinations chosen for the diagonal terms are shown in Table III.

Table III

Group I Computer Runs

Satellite/ Station	Run	diag P(0)	Observations Used	Range (KM) Init	Range (KM) Min	Range (KM) Max	Elevation (Deg) Init	Elevation (Deg) Min	Elevation (Deg) Max
3825/ 349	1	$(10^4, 10^4, 10^4, 10^2, 10^2, 10^2)$	36 Obser. 10.1 sec. apart	4312	4130	4312	44	32	44
	2	$(10^5, 10^5, 10^5, 10^3, 10^3, 10^3)$							
	3	$(10^7, 10^7, 10^7, 10^4, 10^4, 10^4)$							
3826/ 348	4	$(10^2, 10^2, 10^2, 10^1, 10^1, 10^1)$	26 Obser. 10.1 sec. apart	906	519	1314	12	4	25
	5	$(10^3, 10^3, 10^3, 10^2, 10^2, 10^2)$							
	6	$(10^5, 10^5, 10^5, 10^3, 10^3, 10^3)$							
3824/ 348	7	$(10^3, 10^3, 10^3, 10^2, 10^2, 10^2)$	33 Obser. 10.1 sec. apart	1056	740	1802	26	9	41
	8	$(10^5, 10^5, 10^5, 10^3, 10^3, 10^3)$							
3823/ 348	9	$(10^3, 10^3, 10^3, 10^2, 10^2, 10^2)$	44 Obser. 10.1 sec. apart	1363	659	2169	13	2	39
	10	$(10^5, 10^5, 10^5, 10^3, 10^3, 10^3)$							

The results obtained in the first ten computer runs, along with results provided by SPADATS, are shown in Table IV. The approximate time when the orbital elements are computed by SPADATS are from three to eighteen hours different than the time of the observations used in the Group I runs. Radar observations are usually available at the time when SPADATS computes their orbital elements, but they are not of sufficient quantity to allow the computer algorithm to reach steady values for the elements.

The effects upon the linear error estimates caused by using three different values for P(0) for Satellite 3825 are shown in Figs. 4-9. The time history of the diagonal elements and two off-diagonal elements of the error covariance matrix are shown in Figs. 10-15. The resulting orbital elements are plotted versus time in Figs. 16-18. Similar curves for only one value of P(0) are presented in Figs. 19-33 for Satellites 3826, 3824, and 3823. The P(0) chosen is the one which gave the best overall results for each satellite. Additional plots produced by using other initial values for the error covariance matrix are presented in Appendix E.

<u>Analysis of Group I Results</u>

For each satellite, it is possible to obtain convergence of the error estimates to values less than 0.1 per cent of the actual values of position and velocity. The number of observations needed to obtain convergence

Table IV

Group I Results

Satellite	Source of Results	Approximate Time of Computation			Semi-Major Axis (ER)	Eccentricity	Inclination (Deg)	Line of Nodes (Deg)
		Pass	Day	Hour				
3825	SPADATS		78	3	1.4884	0.2789	104.815	342.70
3825	Run 1	12	78	21	1.4905	0.2796	104.950	343.52
3825	Run 2	12	78	21	1.4955	0.2773	104.732	343.10
3825	Run 3	12	78	21	1.4968	0.2786	104.750	343.17
3826	SPADATS		78	7	1.0445	0.0159	99.051	343.76
3826	Run 4	2	78	10	1.0457	0.0141	99.051	344.36
3826	Run 5	2	78	10	1.0448	0.0150	99.040	344.30
3826	Run 6	2	78	10	1.0451	0.0155	99.039	344.32
3824	SPADATS		77	12	1.0822	0.0076	98.879	343.24
3824	Run 7	1	77	9	1.0803	0.0085	98.897	343.04
3824	Run 8	1	77	9	1.0830	0.0079	98.889	342.99
3823	SPADATS		78	3	1.0680	0.0049	99.186	344.00
3823	Run 9	8	78	10	1.0683	0.0086	99.180	344.34
3823	Run 10	8	78	10	1.0668	0.0070	99.189	344.34

varies from a minimum of 10 (100 seconds) to a maximum of 24 (240 seconds). The rate of convergence is related to the initial estimate of the error covariance matrix, along with the measurement accuracy of the tracking stations. Underestimating P(0) delays convergence, which in turn prevents the orbital elements from reaching steady values. Overestimating P(0) greatly improves the rate of convergence, and this also results in very steady orbital elements in the minimum time.

The major discrepancy in the Group I results is the value obtained for the semi-major axis for Satellite 3825 as compared to the value provided by SPADATS. The measurements for Satellite 3825 are obtained from Station 349, and these measurements contain larger bias errors and sigmas (standard deviations) than any of the measurements from the other tracking stations used, as shown in Table I. This fact provides a possible explanation for the discrepancy in the answers obtained by using data from Station 349.

The Line of Nodes angle had a positive time rate of change which is a function of the inclination angle, and eccentricity. This rate of change varies from 0.08 to 0.2 degrees per revolution for the four satellites, and accounts for some of the discrepancy between the computed values of the Line of Nodes and those values provided by SPADATS.

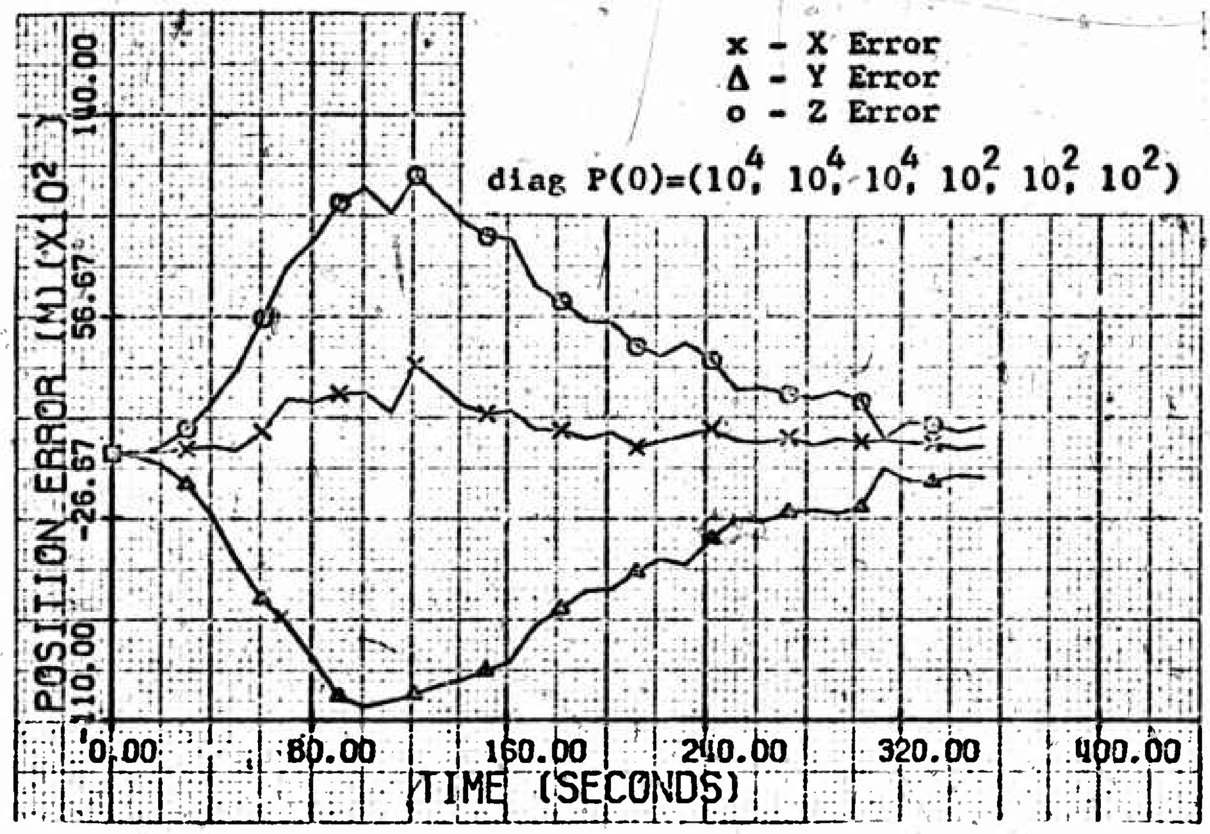

Fig.4 Satellite 3825 Station 349 Pass #12

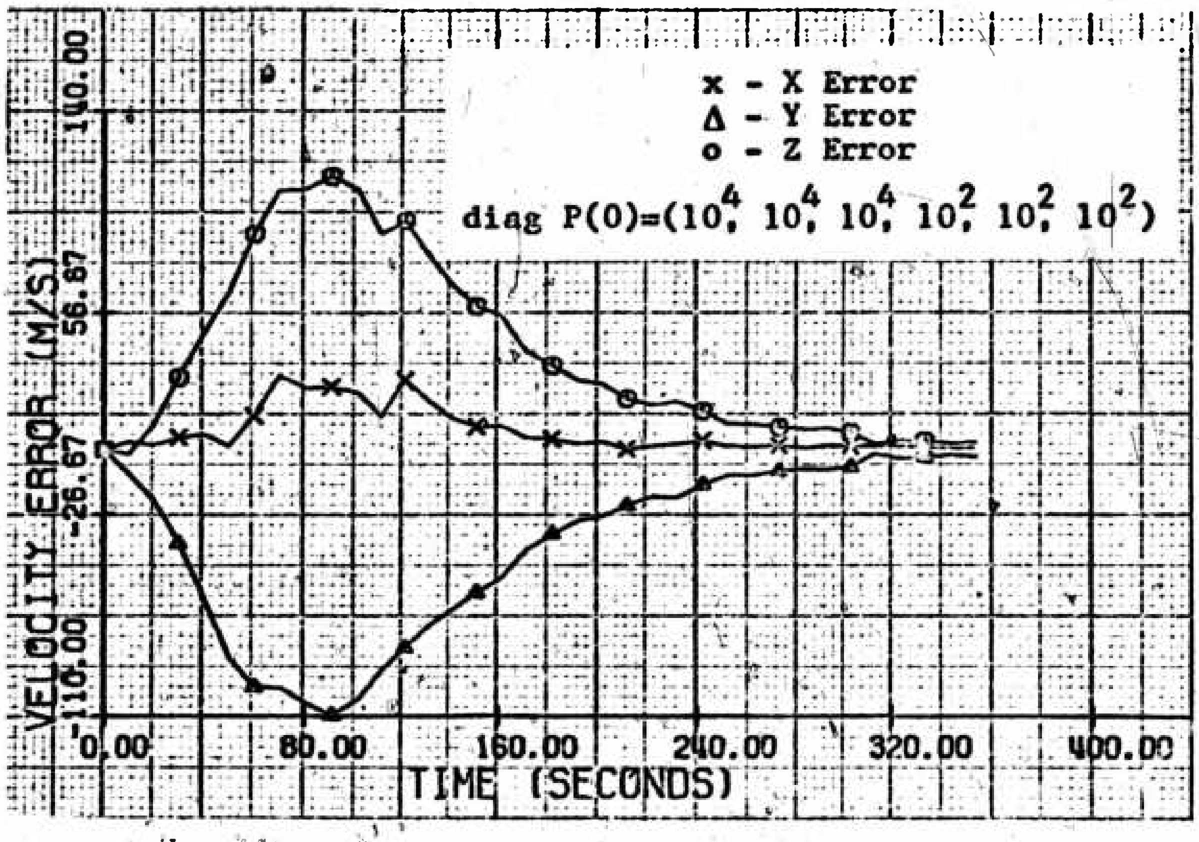

Fig. 5 Satellite 3825 Station 349 Pass #12

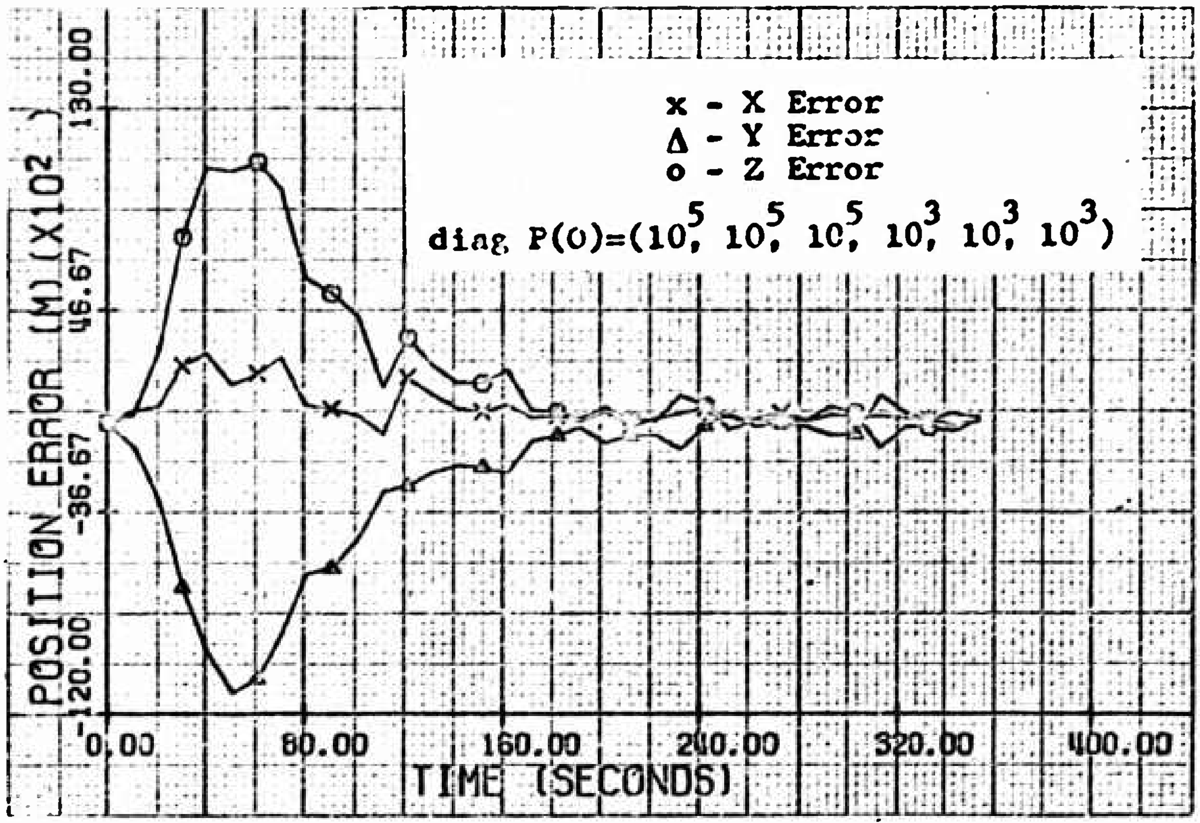

Fig. 6 Satellite 3825 Station 349 Pass #12

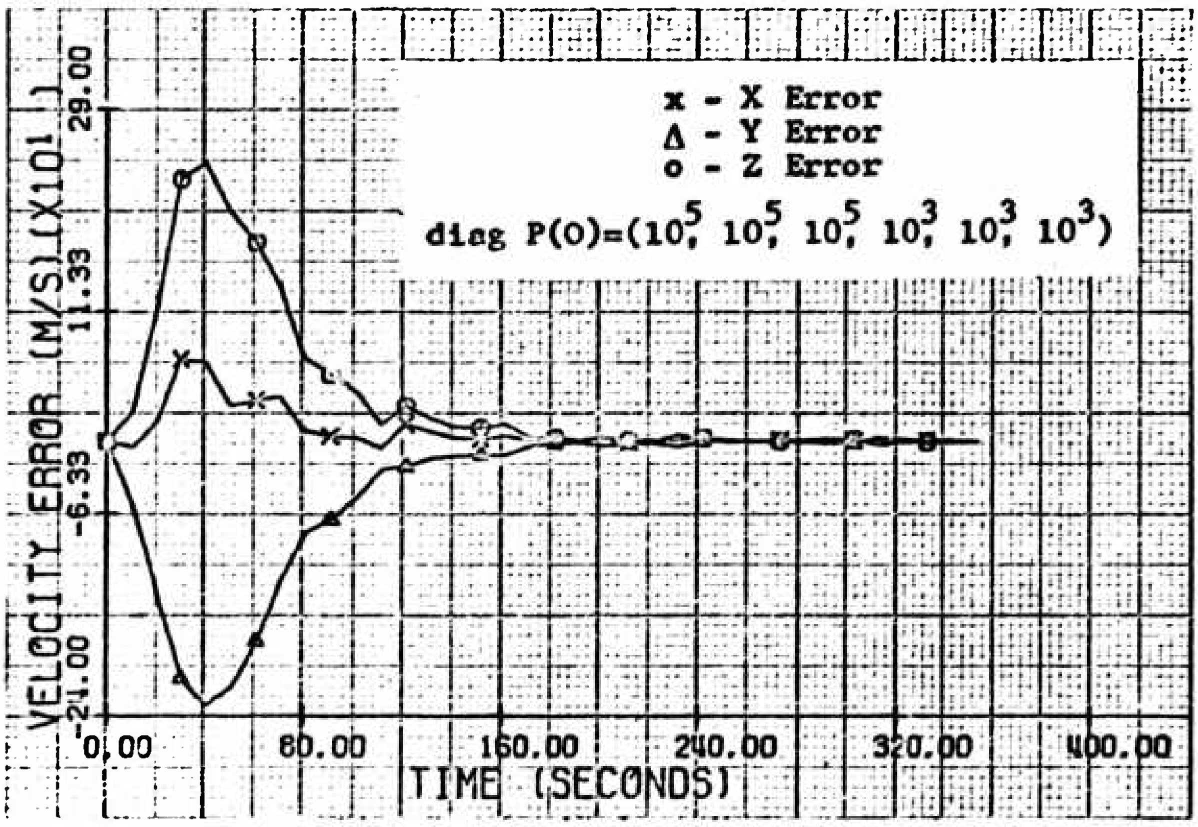

Fig. 7 Satellite 3825 Station 349 Pass #12

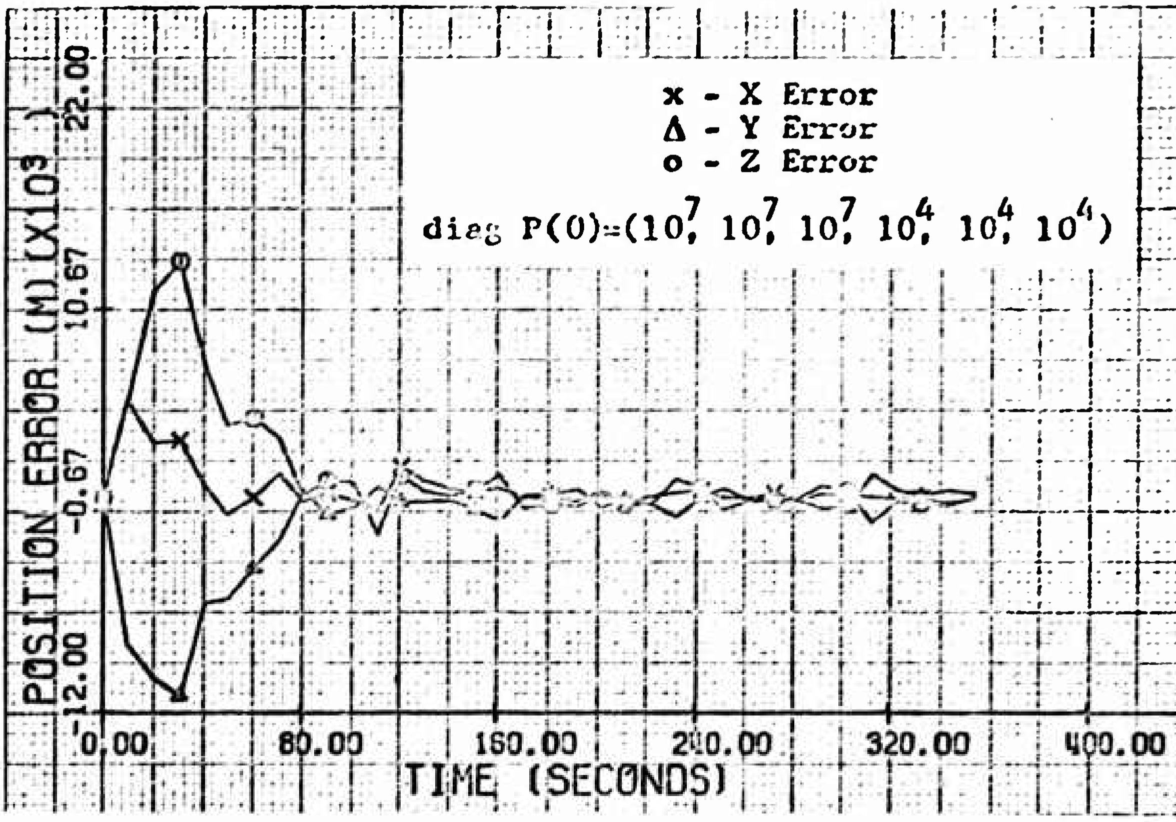

Fig. 8 Satellite 3825 Station 349 Pass #12

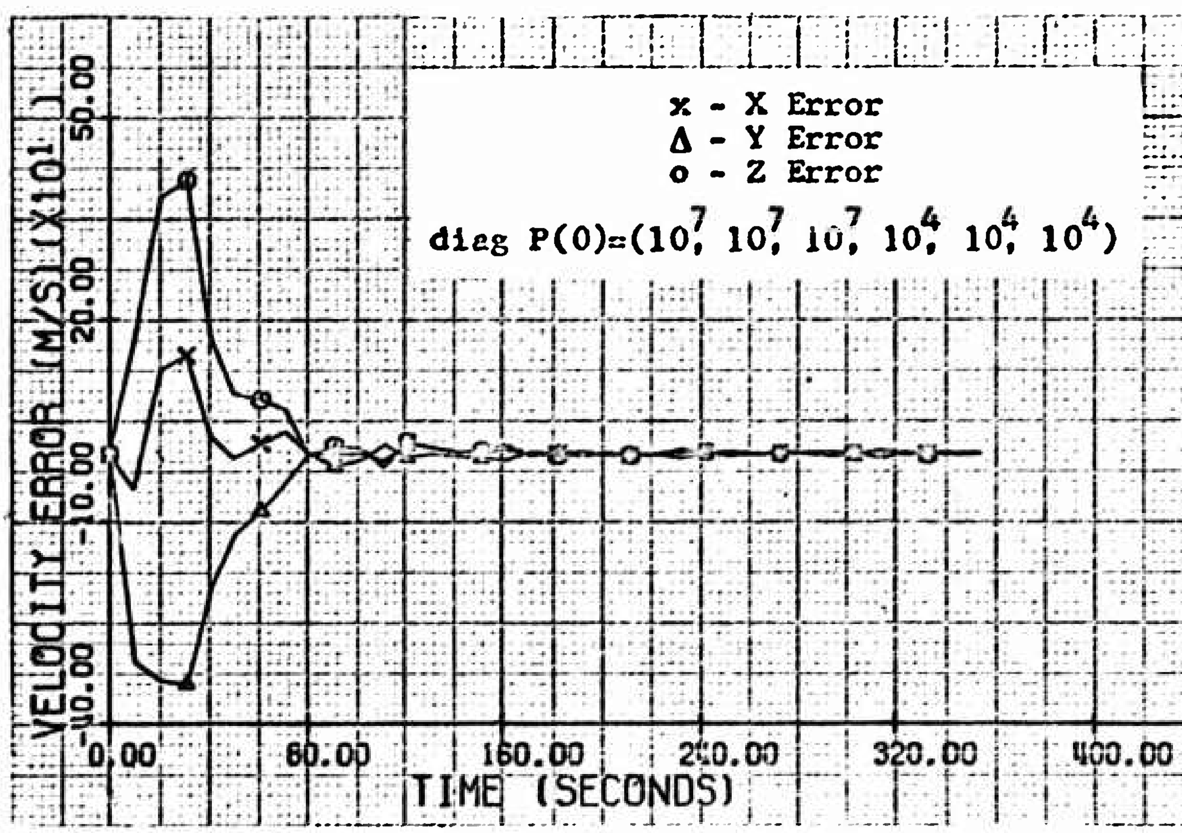

Fig. 9 Satellite 3825 Station 349 Pass #12

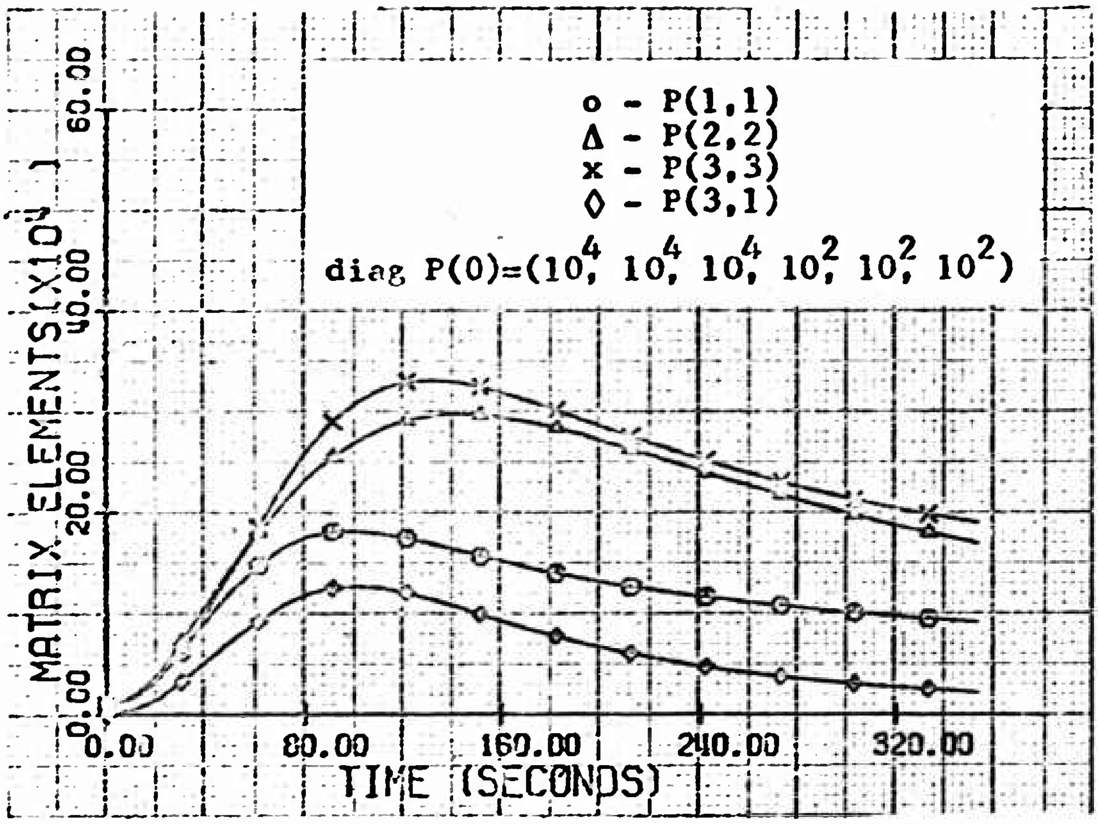

Fig. 10 Satellite 3825 Station 349 Pass #12

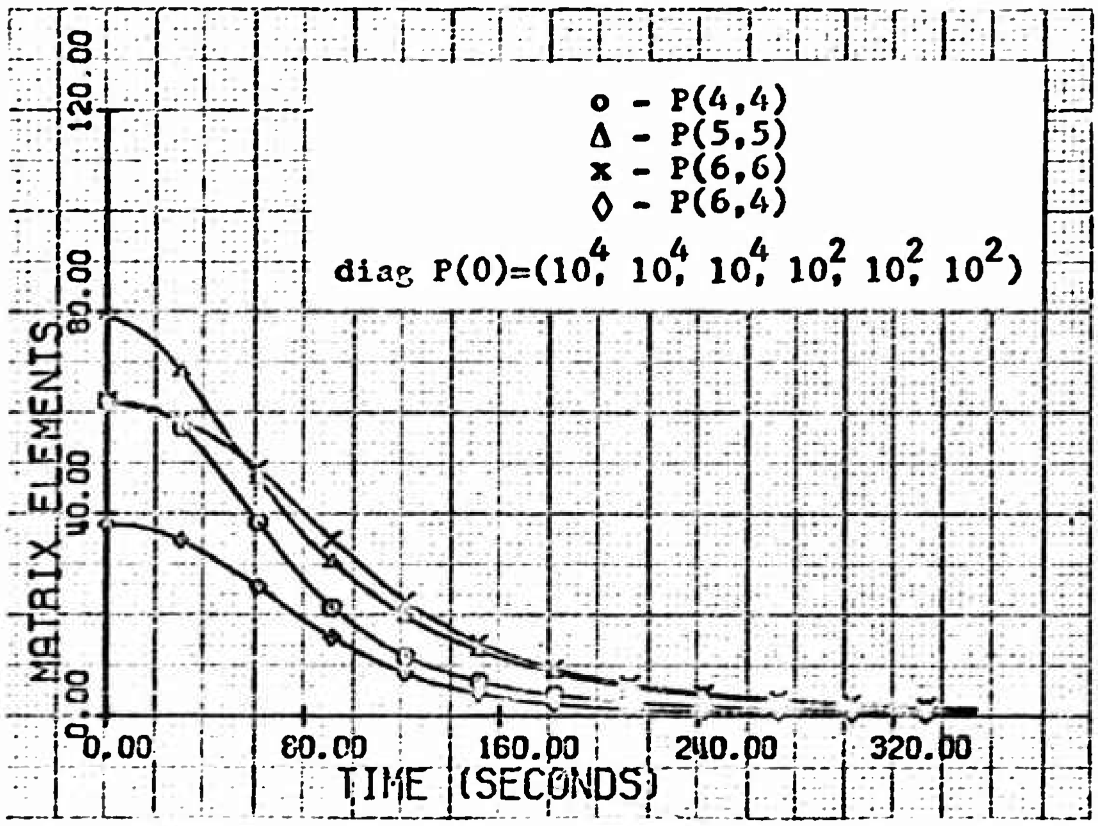

Fig. 11 Satellite 3825 Station 349 Pass #12

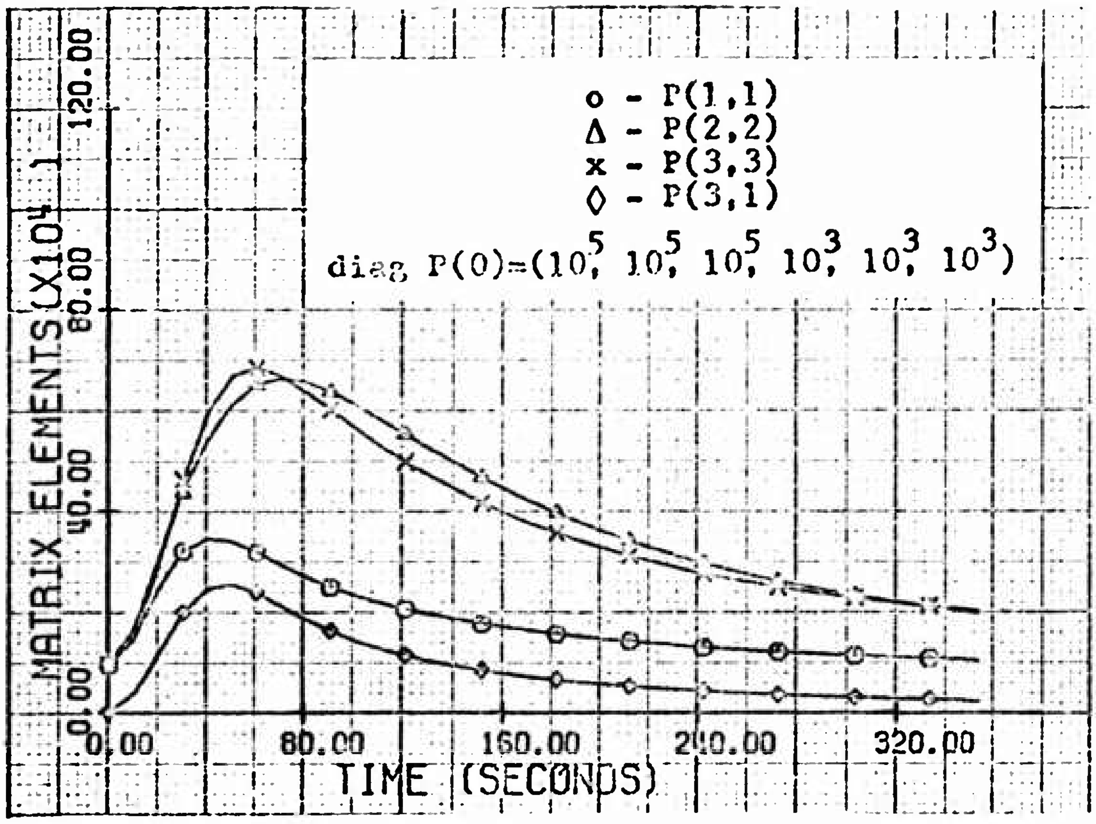

Fig. 12 Satellite 3825 Station 349 Pass #12

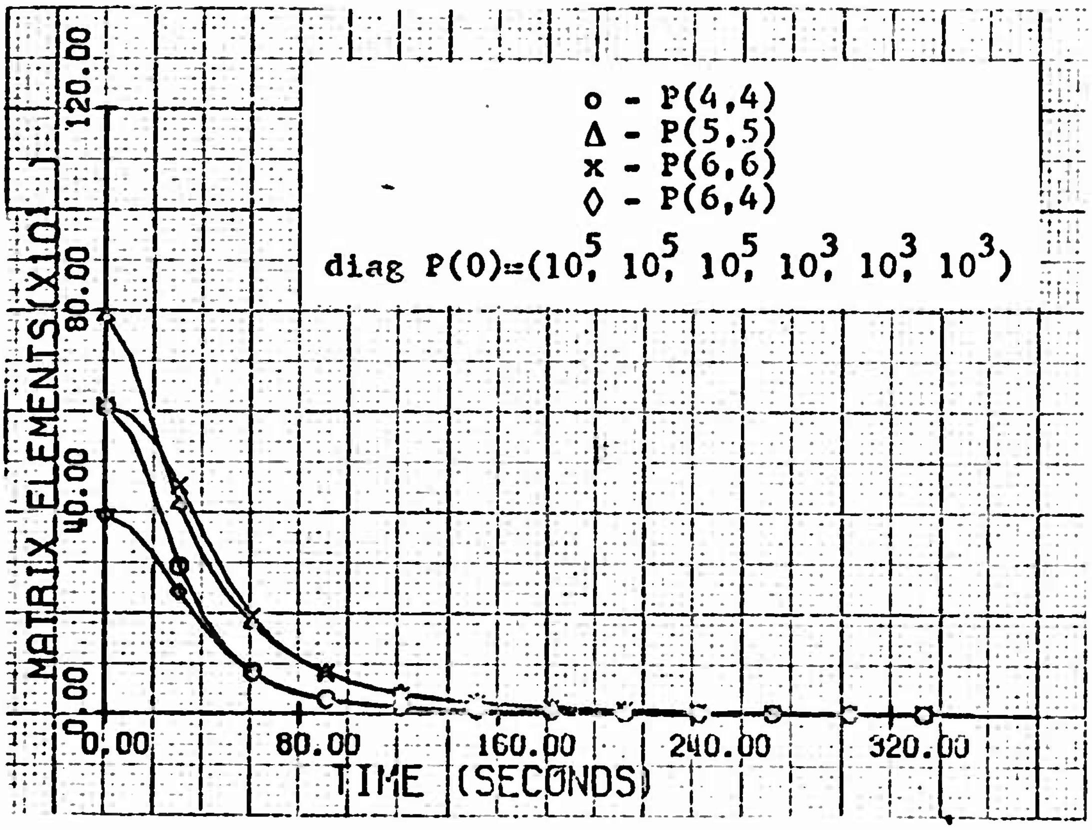

Fig. 13 Satellite 3825 Station 349 Pass #12

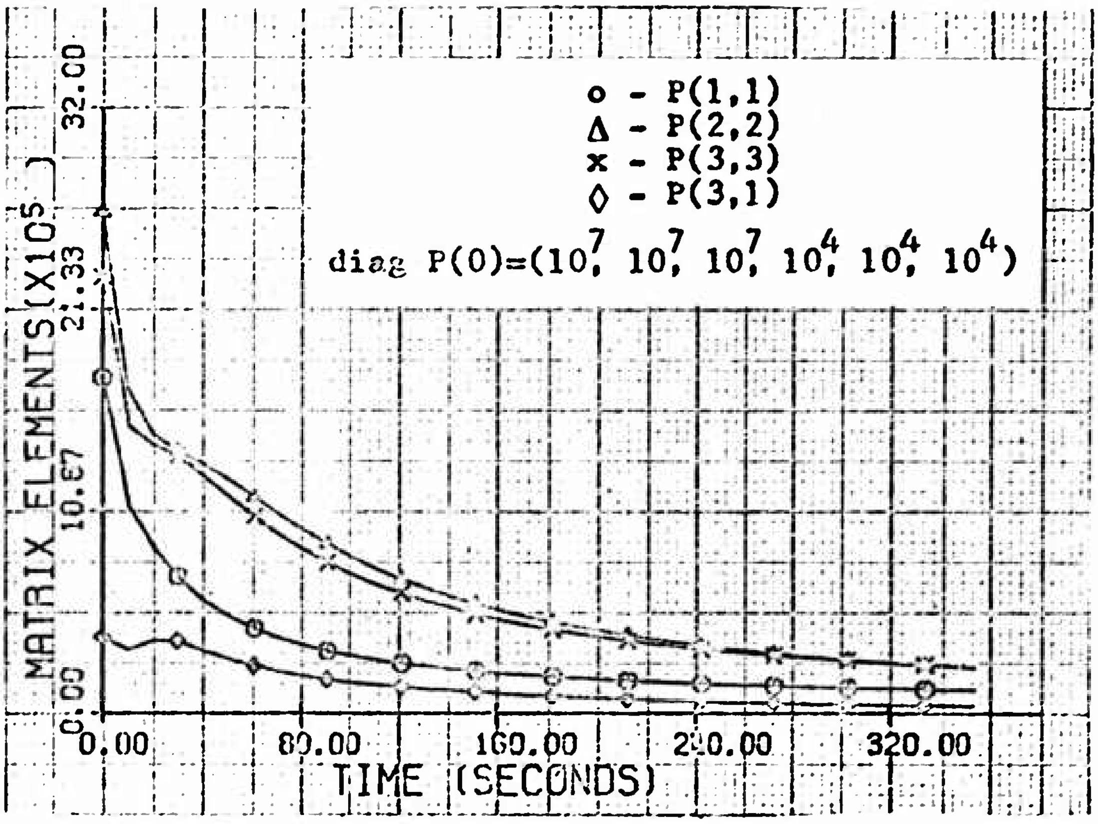

Fig. 14 Satellite 3825 Station 349 Pass #12

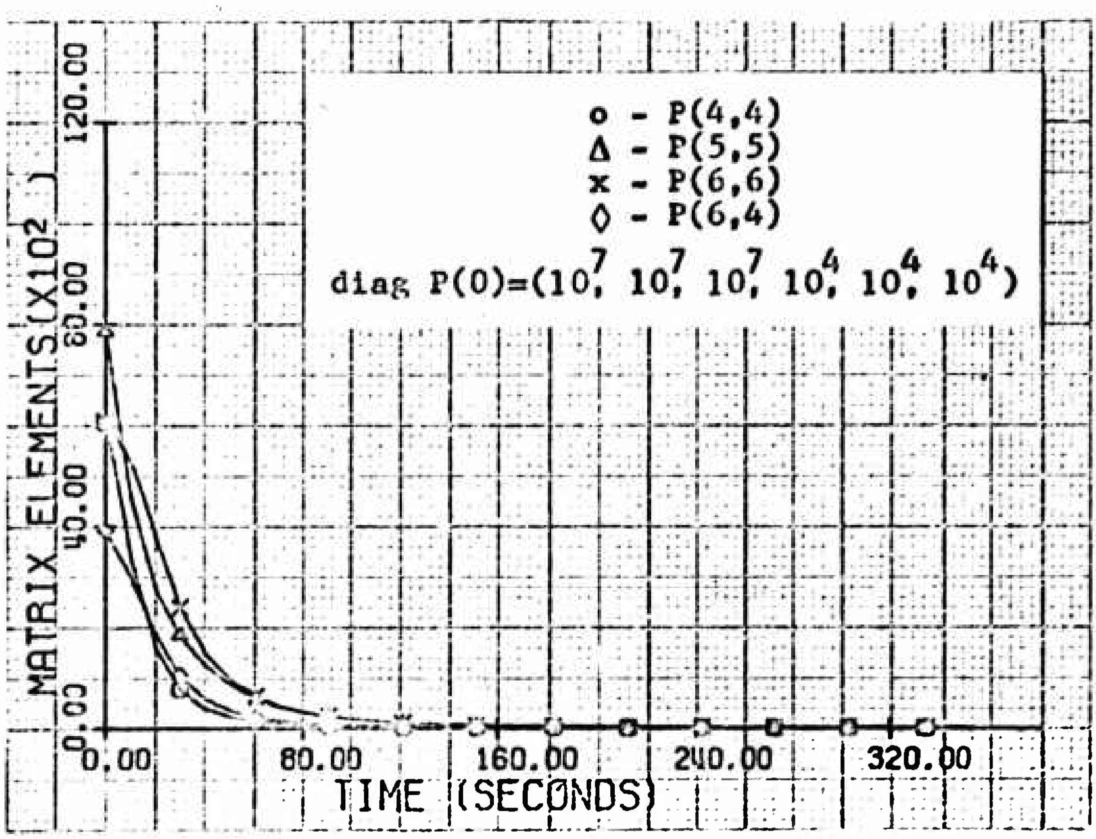

Fig. 15 Satellite 3825 Station 349 Pass #12

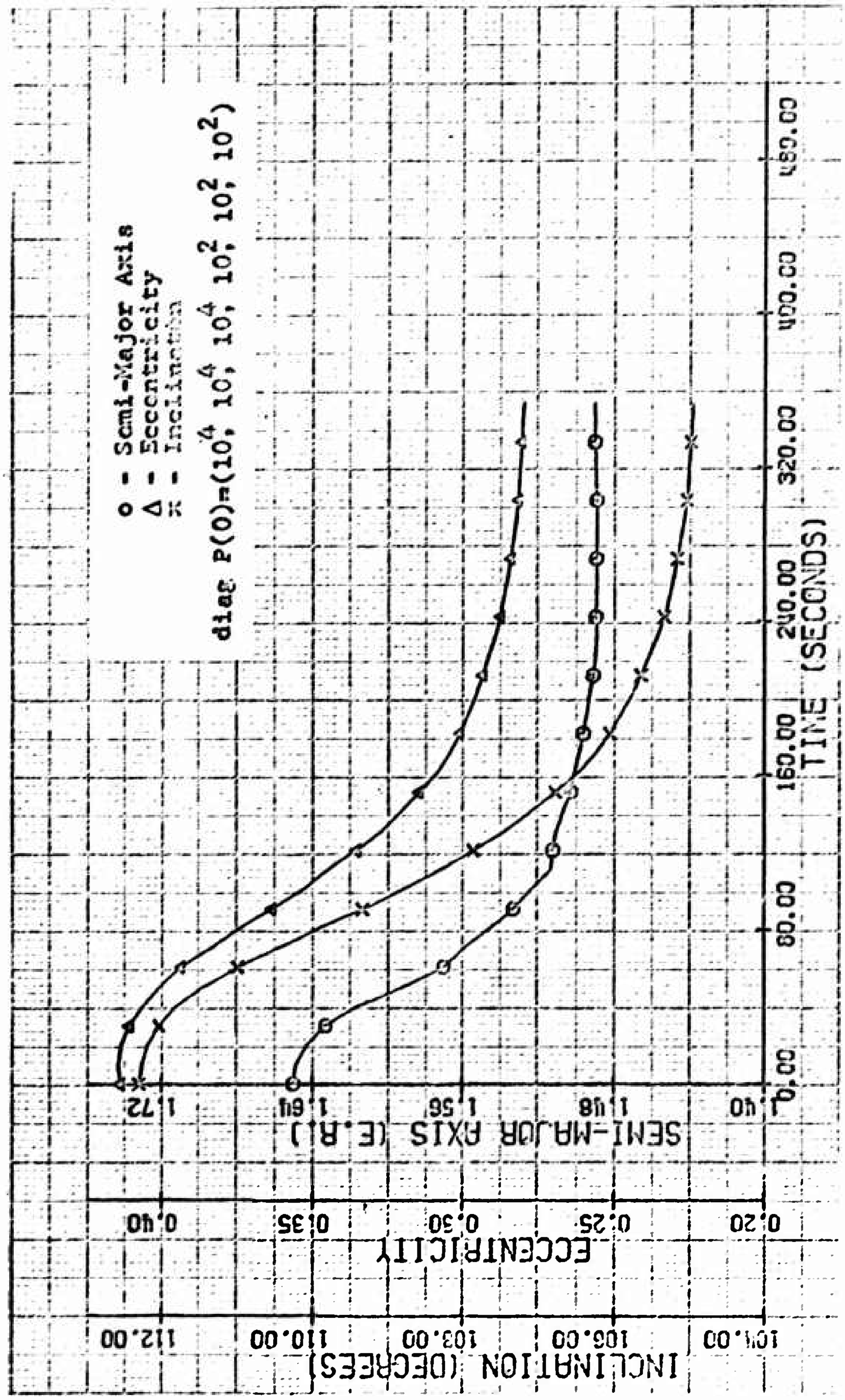

Fig. 16 Satellite 3825 Station 349 Pass #12

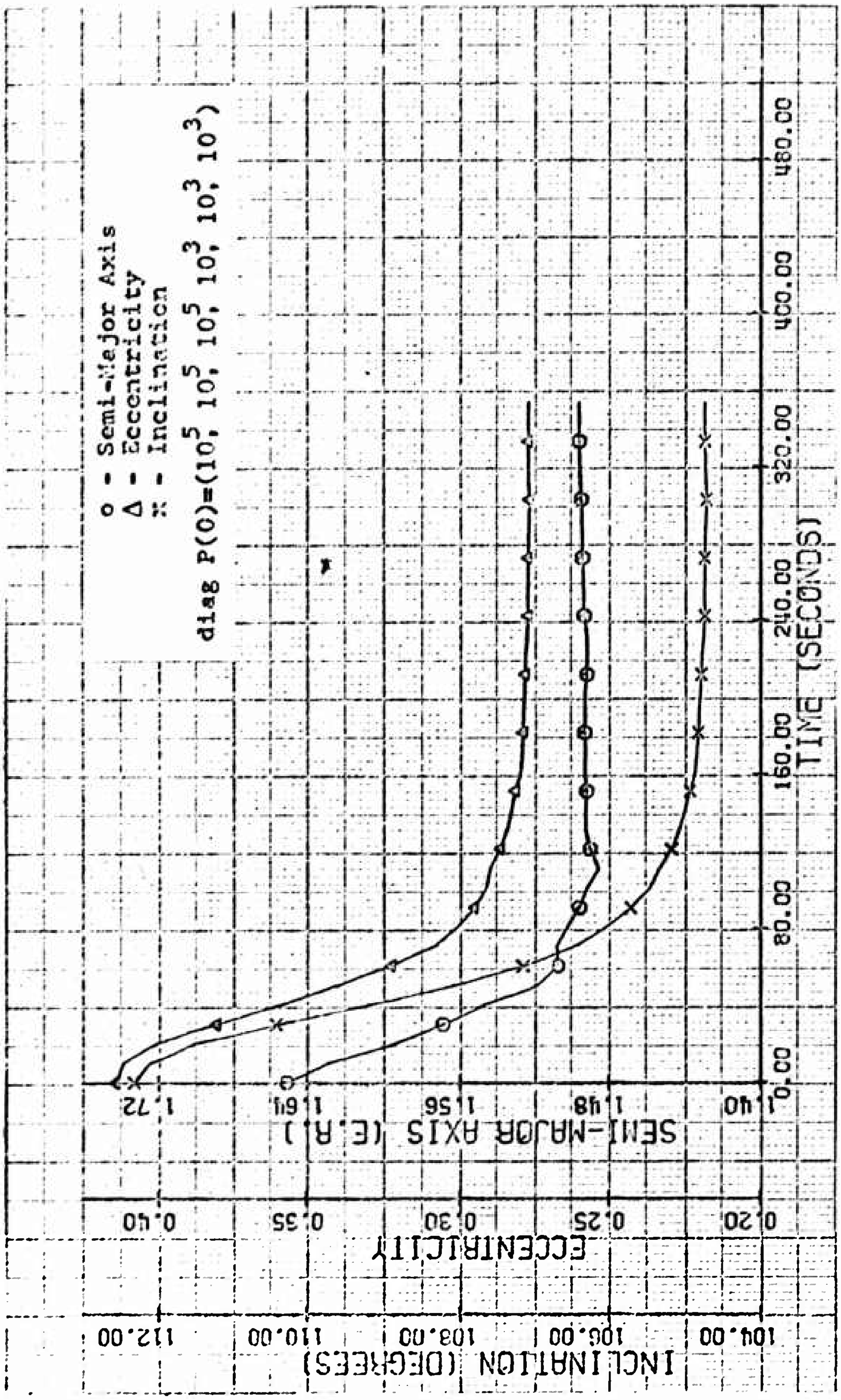

Fig. 17 Satellite 3825 Station 349 Pass #12

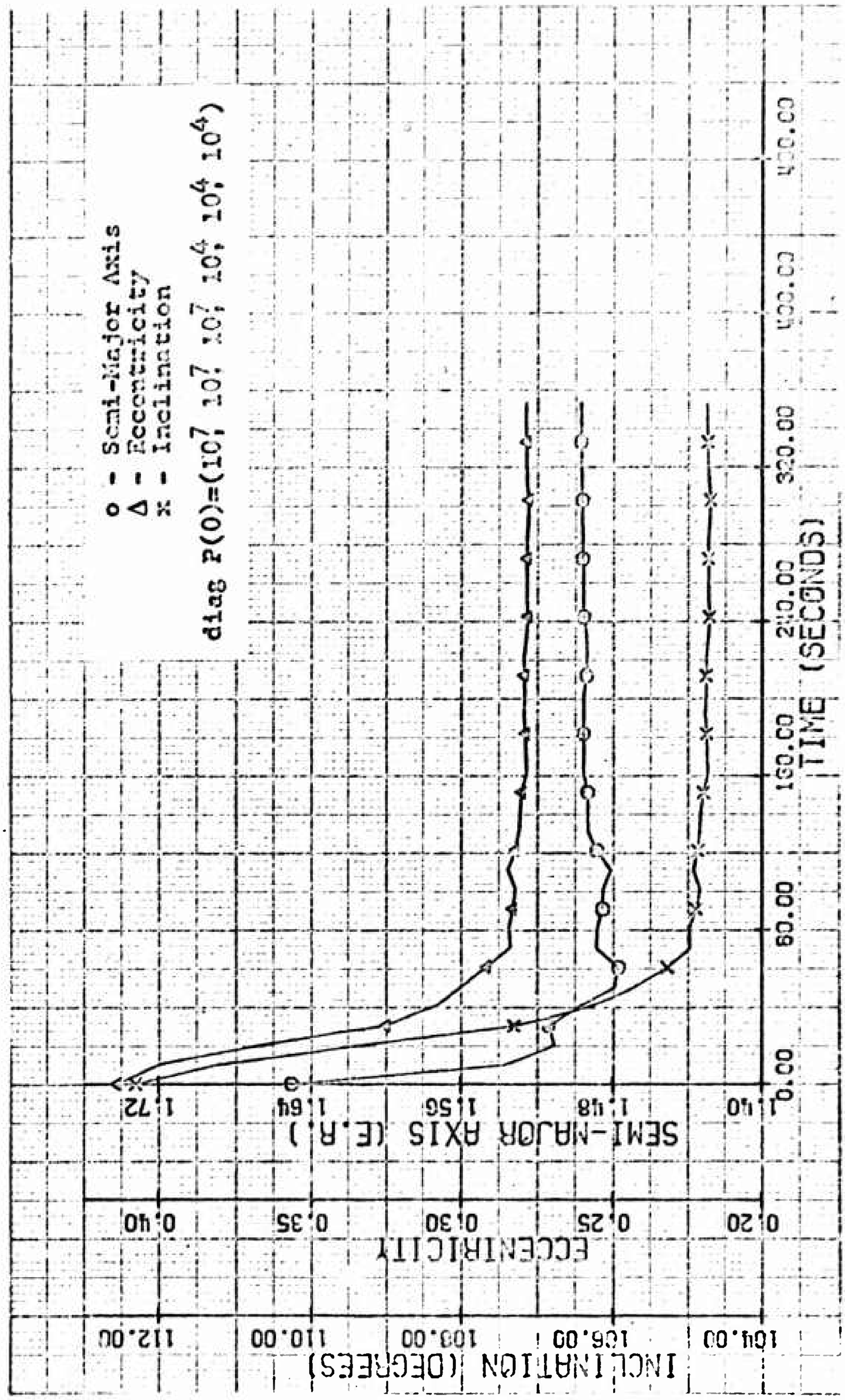

Fig. 18 Satellite 3825 Station 349 Pass #12

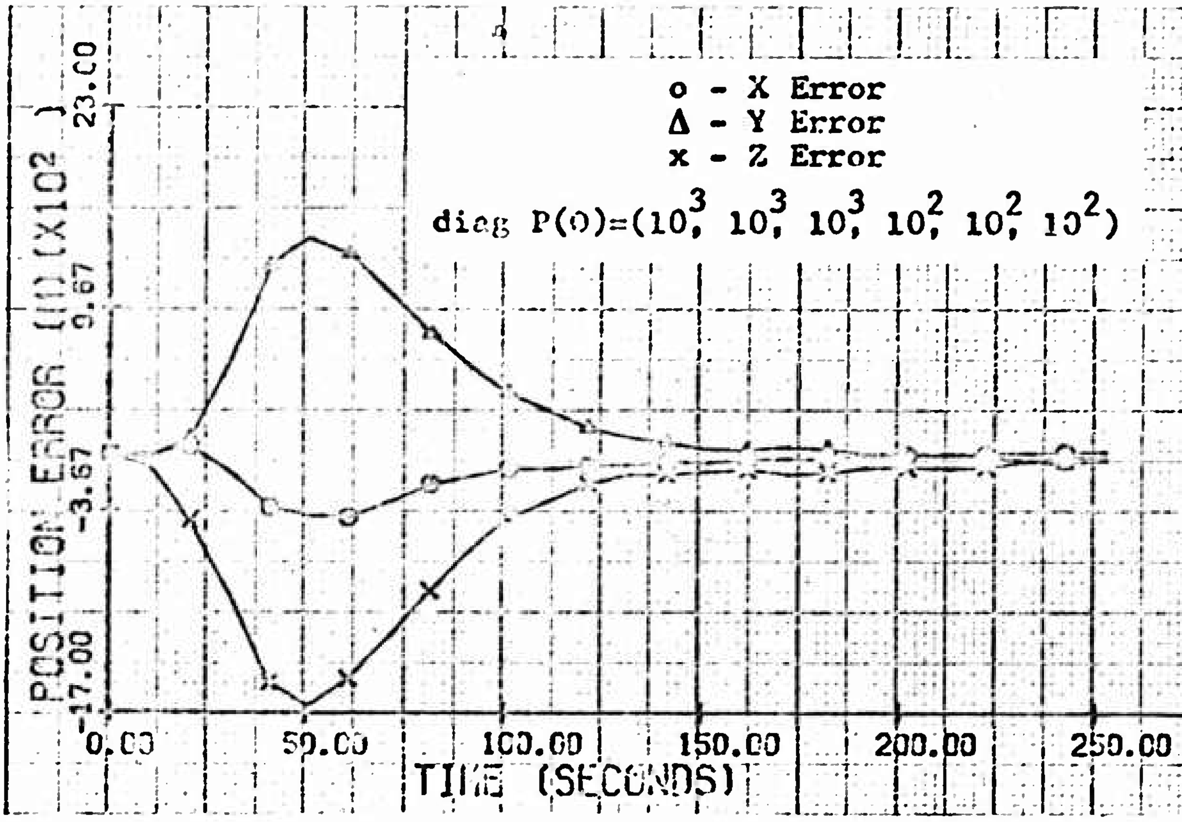

Fig. 19 Satellite 3826 Station 348 Pass #2

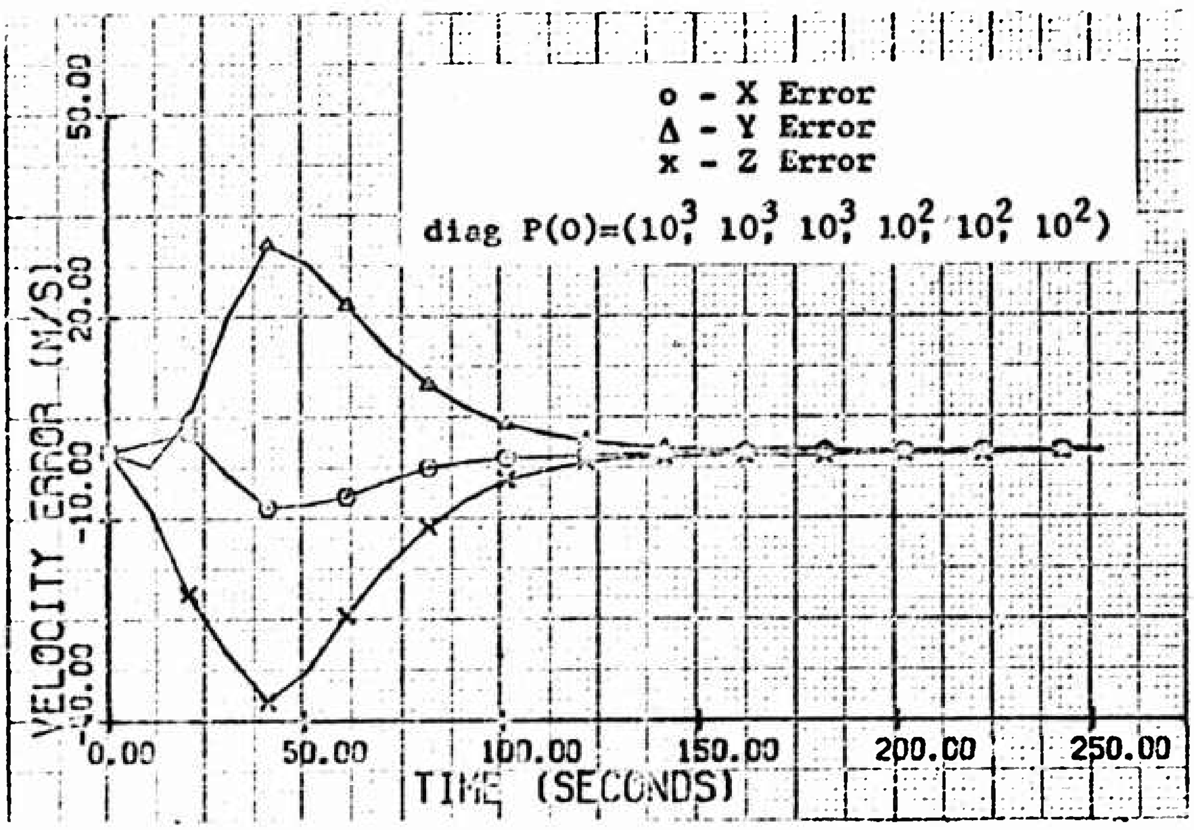

Fig. 20 Satellite 3826 Station 348 Pass #2

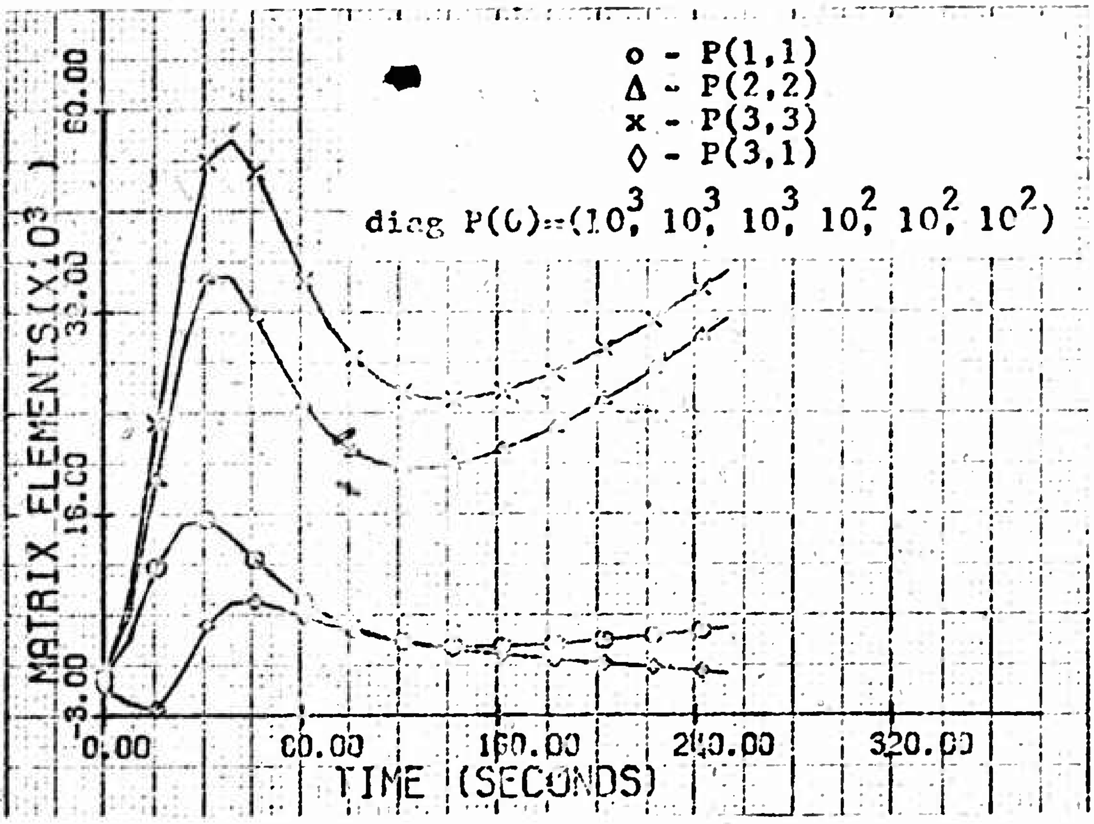

Fig. 21 Satellite 3826 Station 348 Pass #2

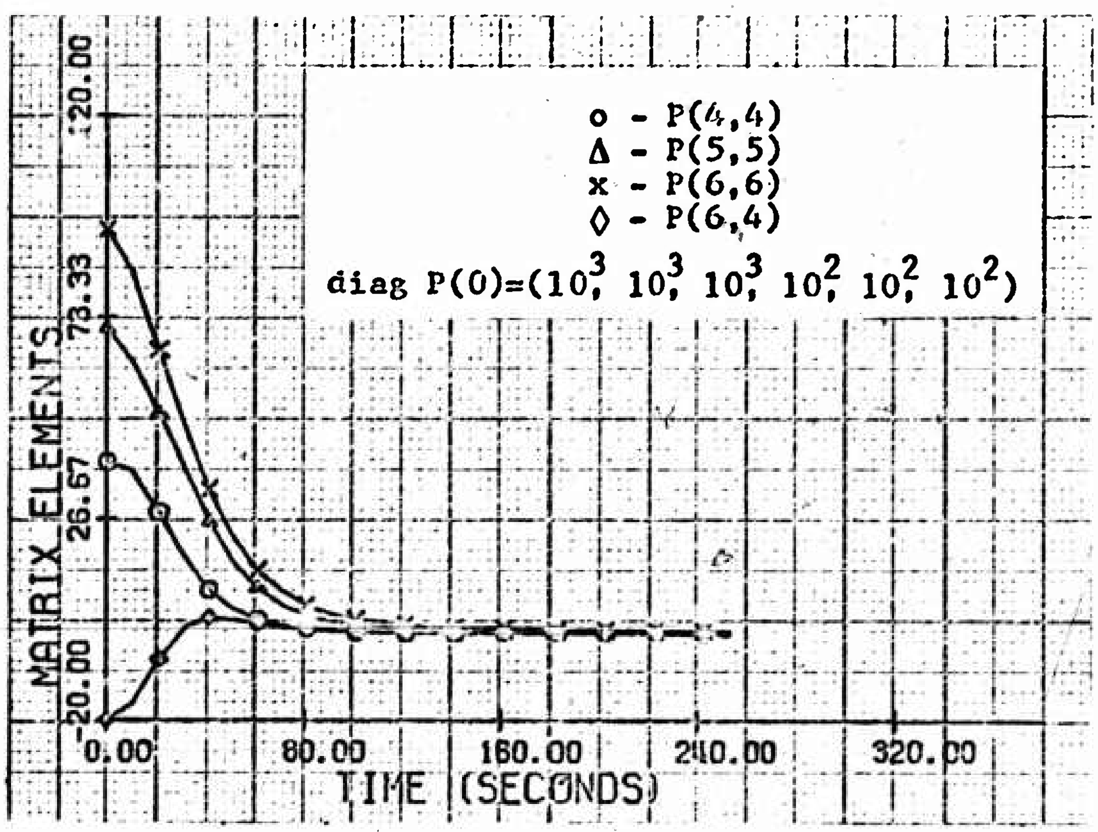

Fig. 22 Satellite 3826 Station 348 Pass #2

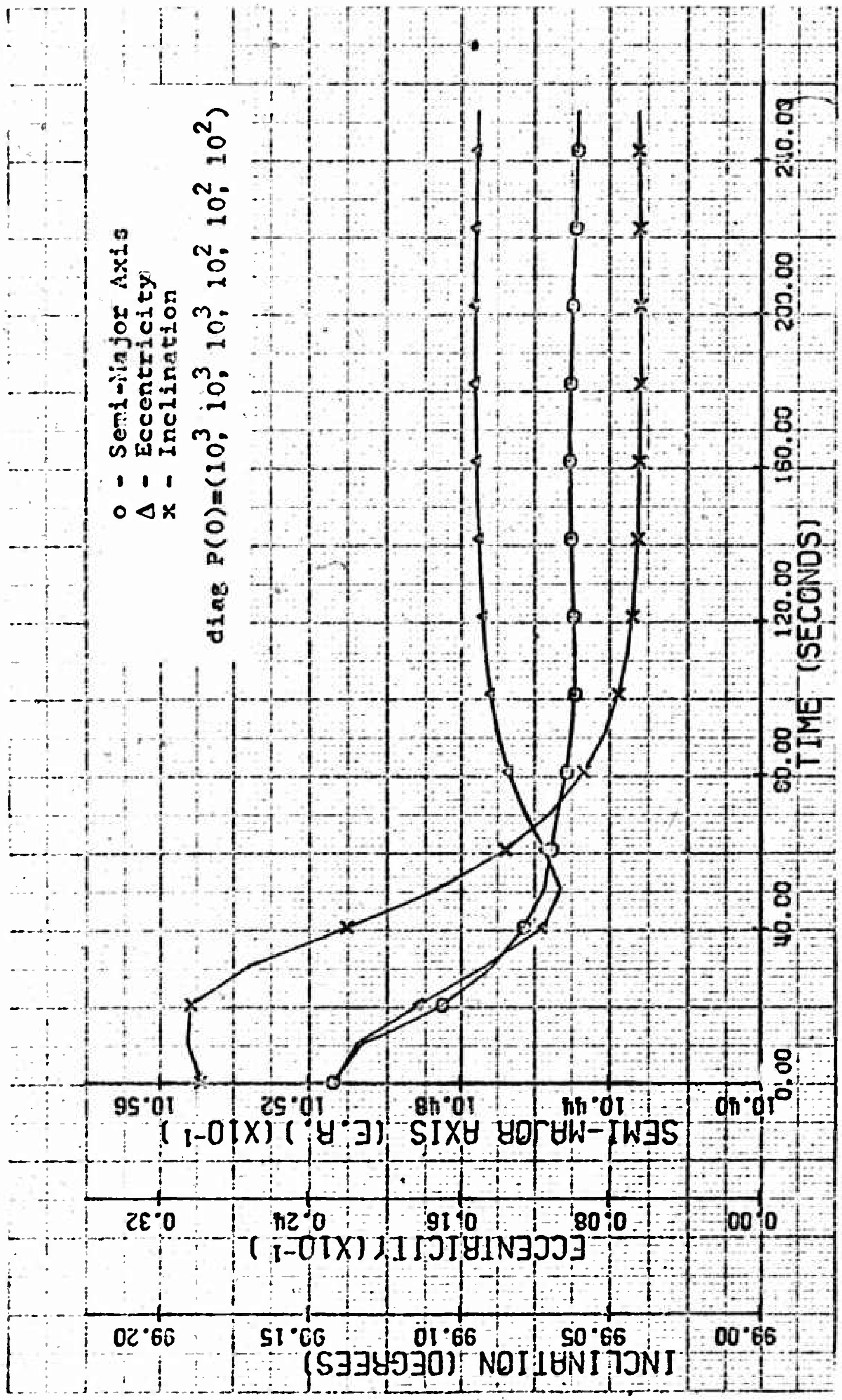

Fig. 23 Satellite 3826 Station 348 Pass #2

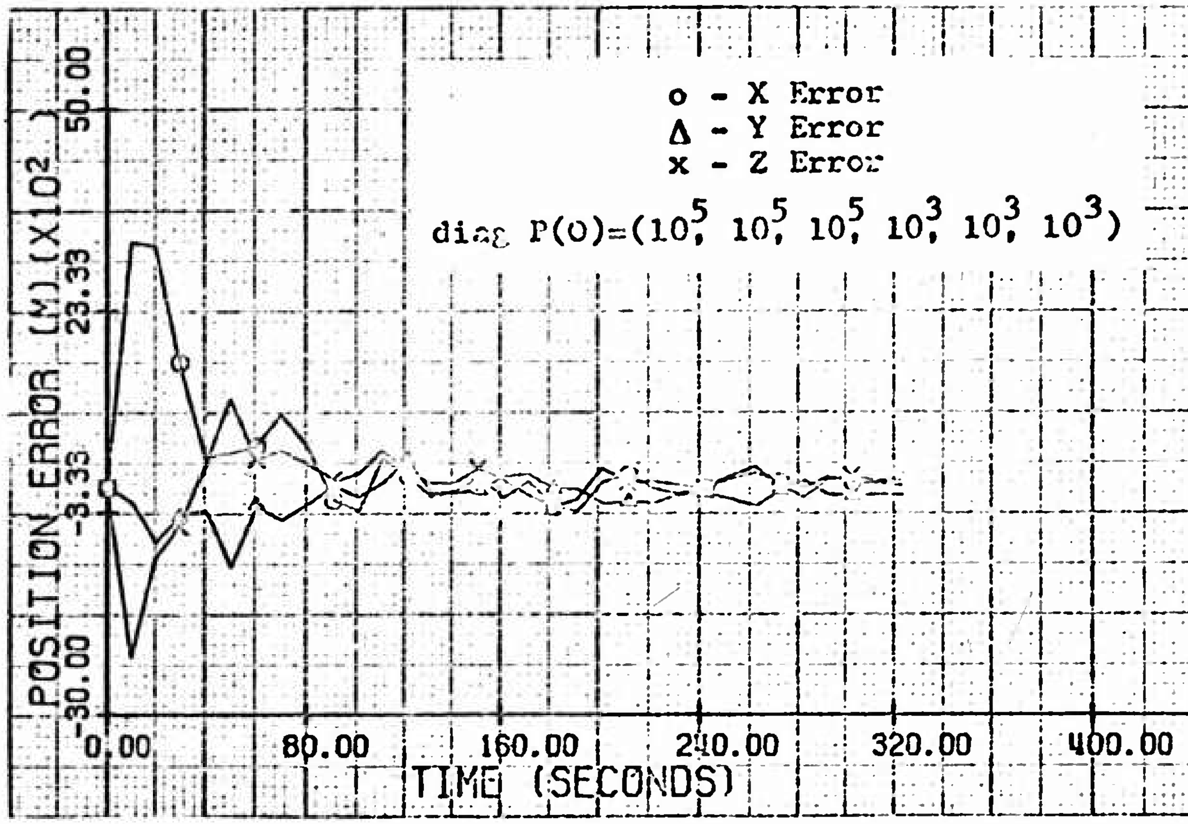

Fig. 24 Satellite 3824 Station 348 Pass #1

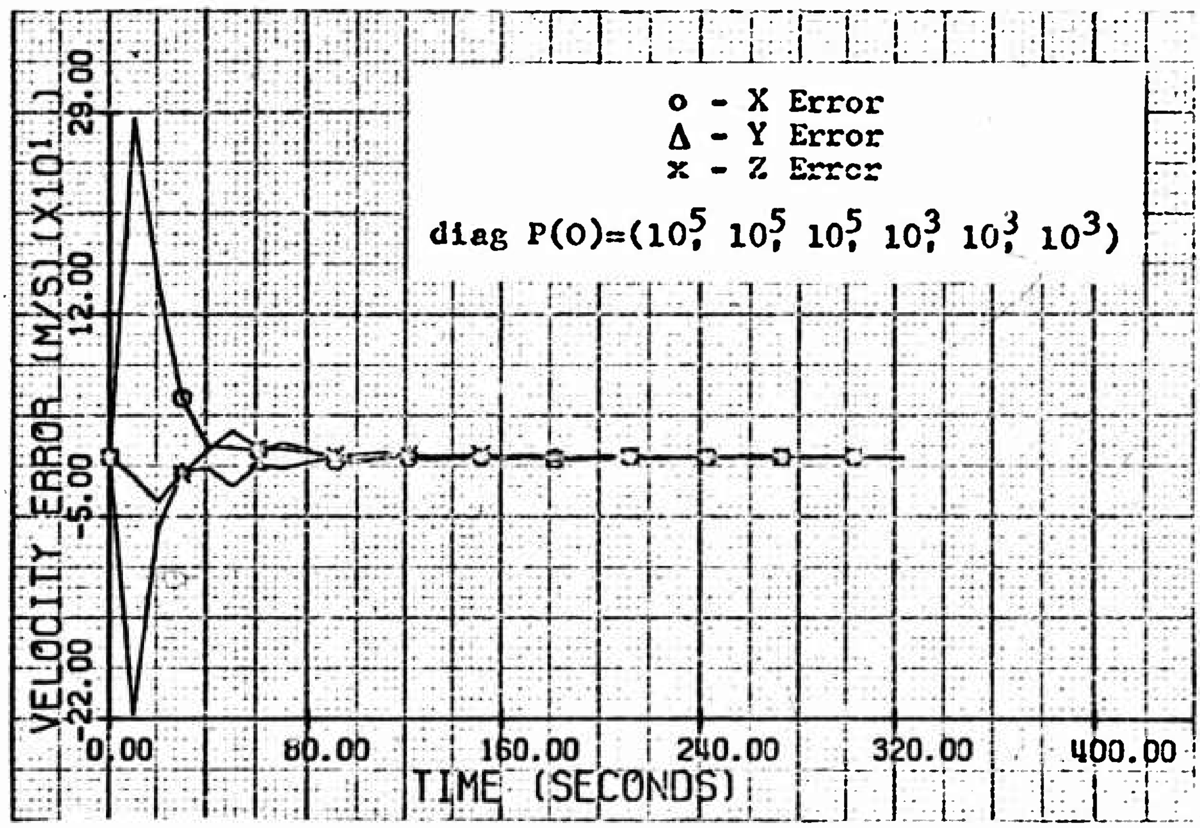

Fig. 25 Satelli'e 3824 Station 348 Pass #1

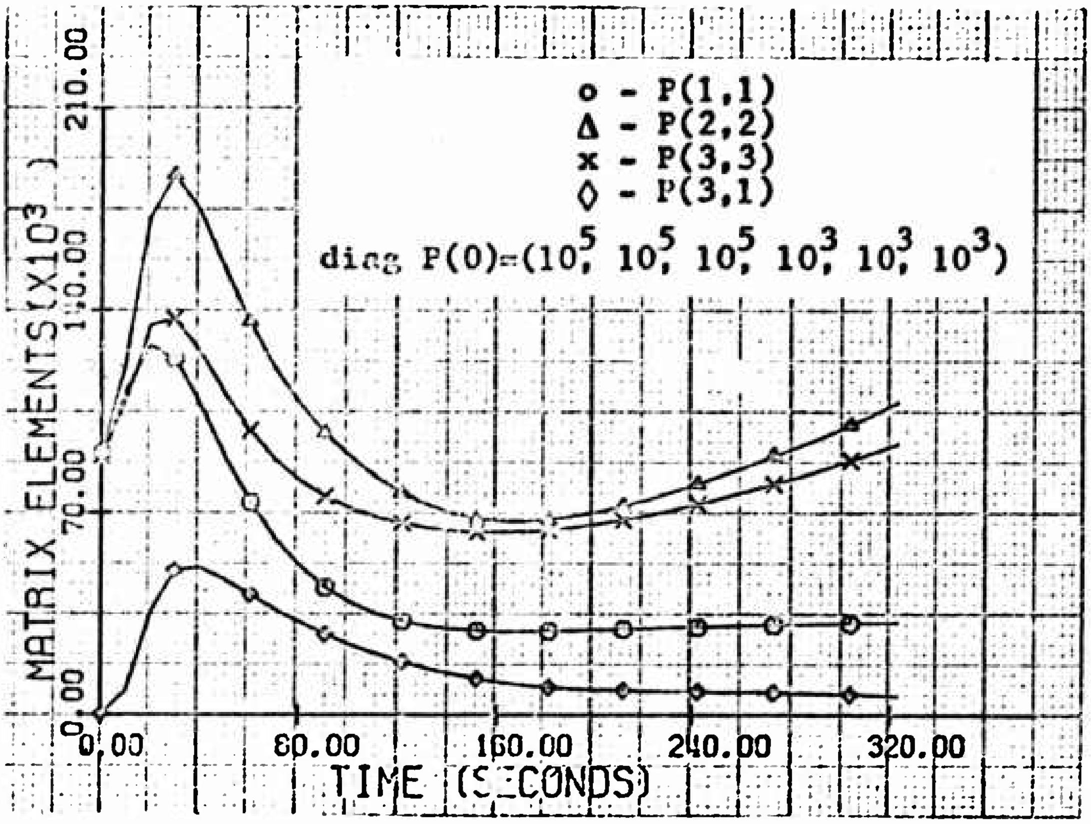

Fig. 26 Satellite 3824 Station 348 Pass #1

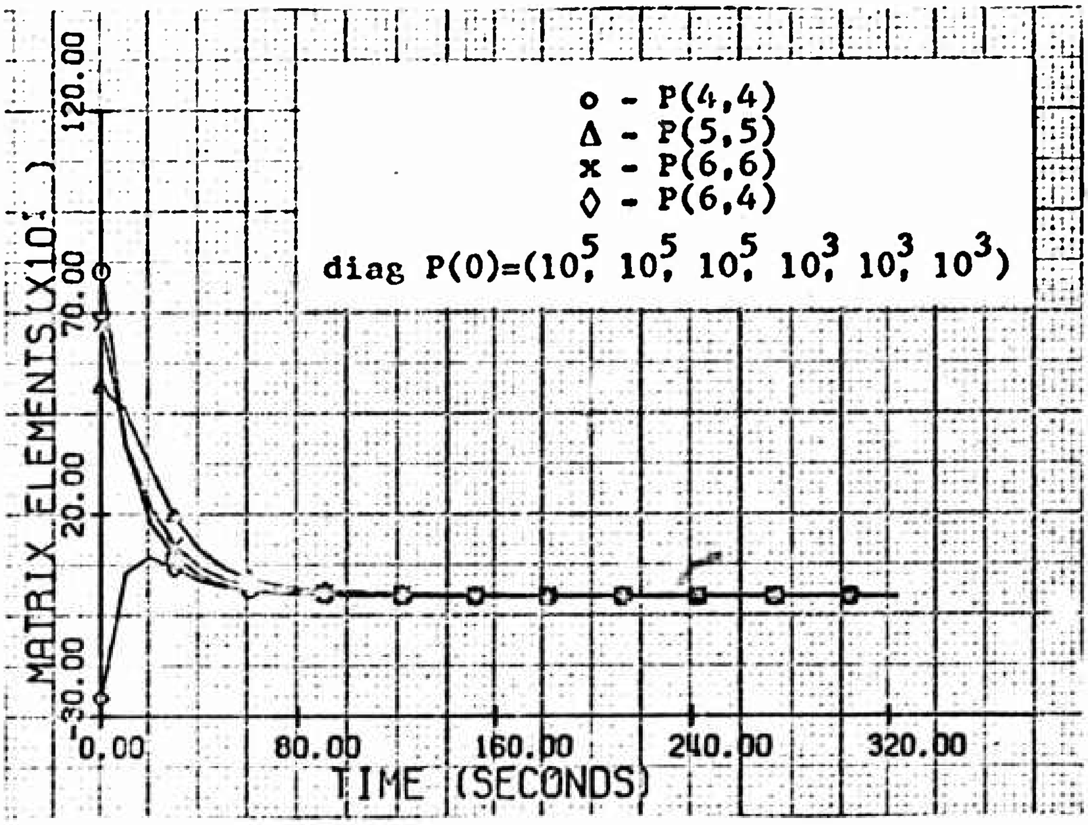

Fig. 27 Satellite 3824 Station 348 Pass #1

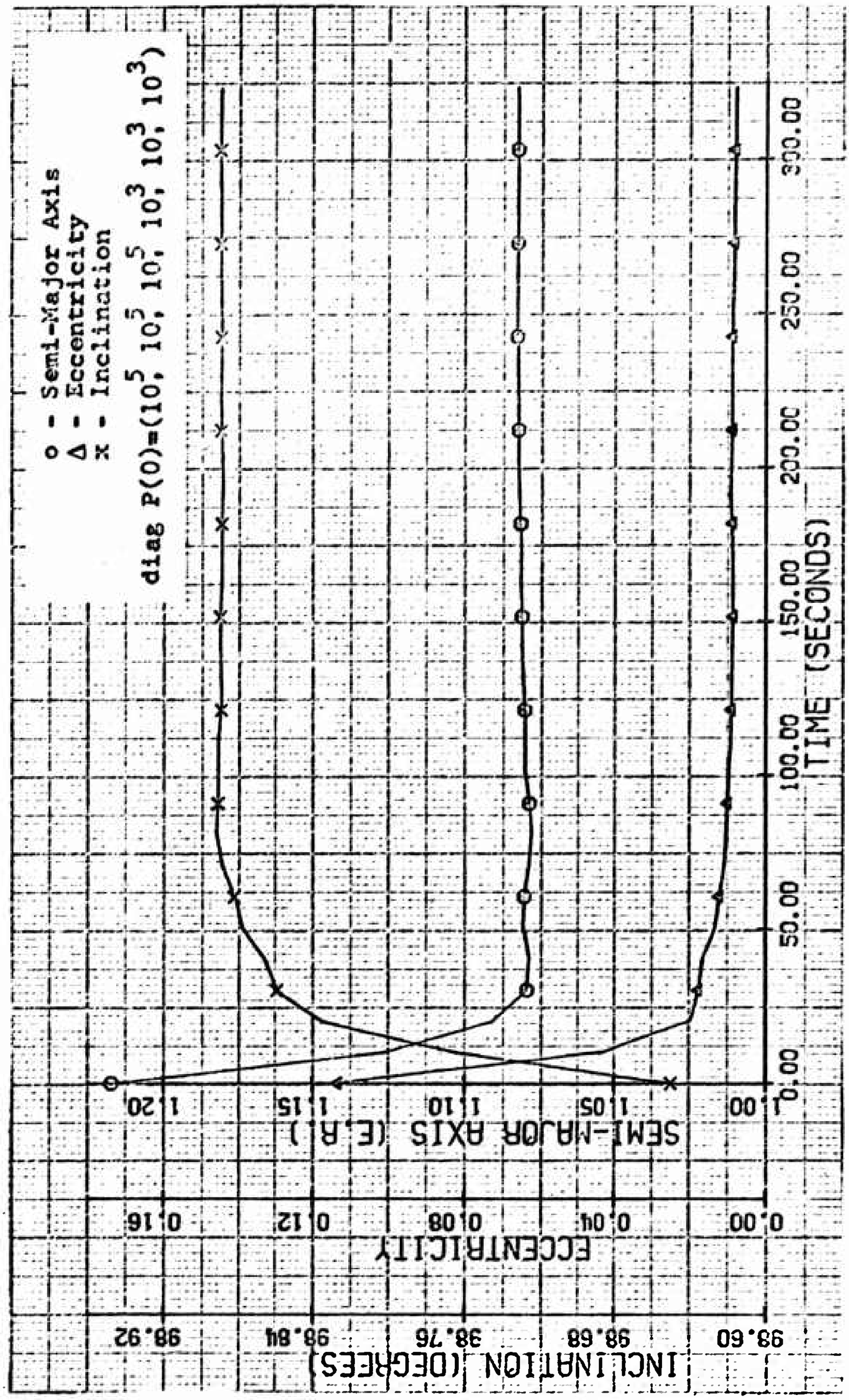

Fig. 28 Satellite 3824 Station 348 Pass #1

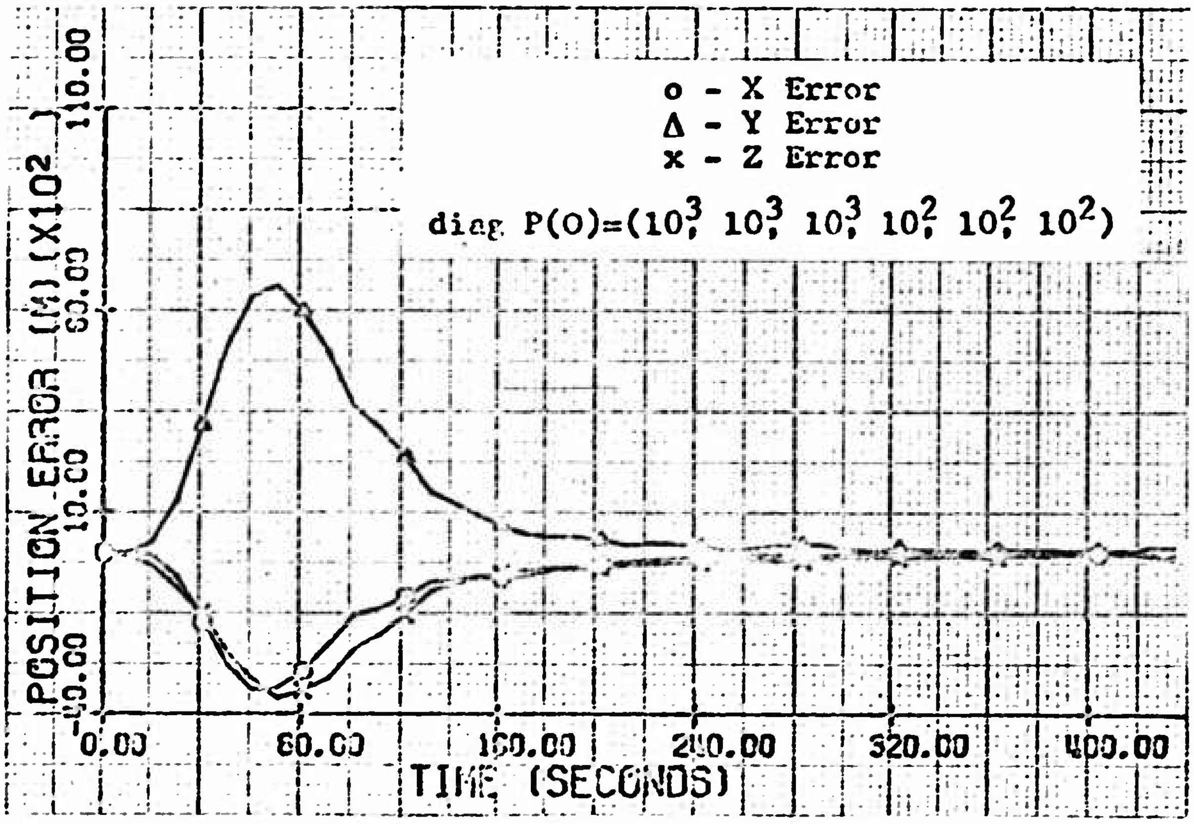

Fig. 29 Satellite 3823 Station 348 Pass #8

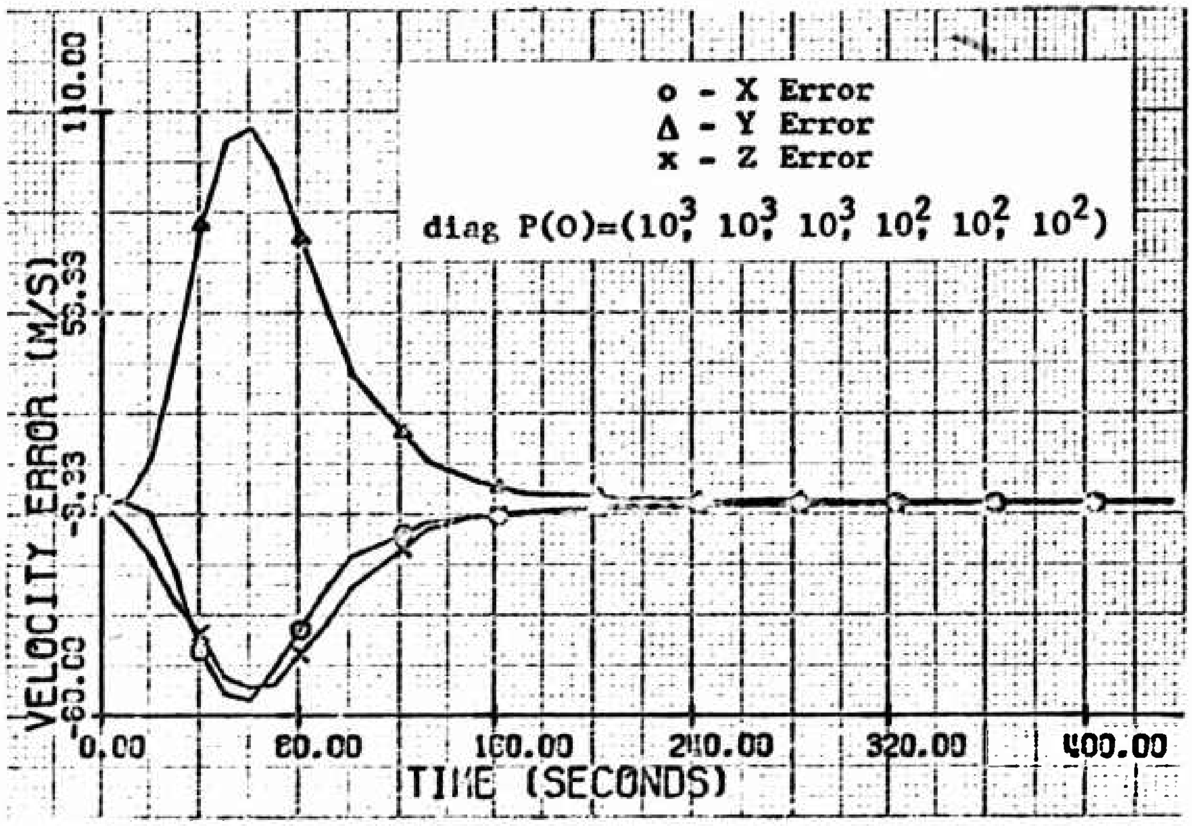

Fig. 30 Satellite 3823 Station 348 Pass #8

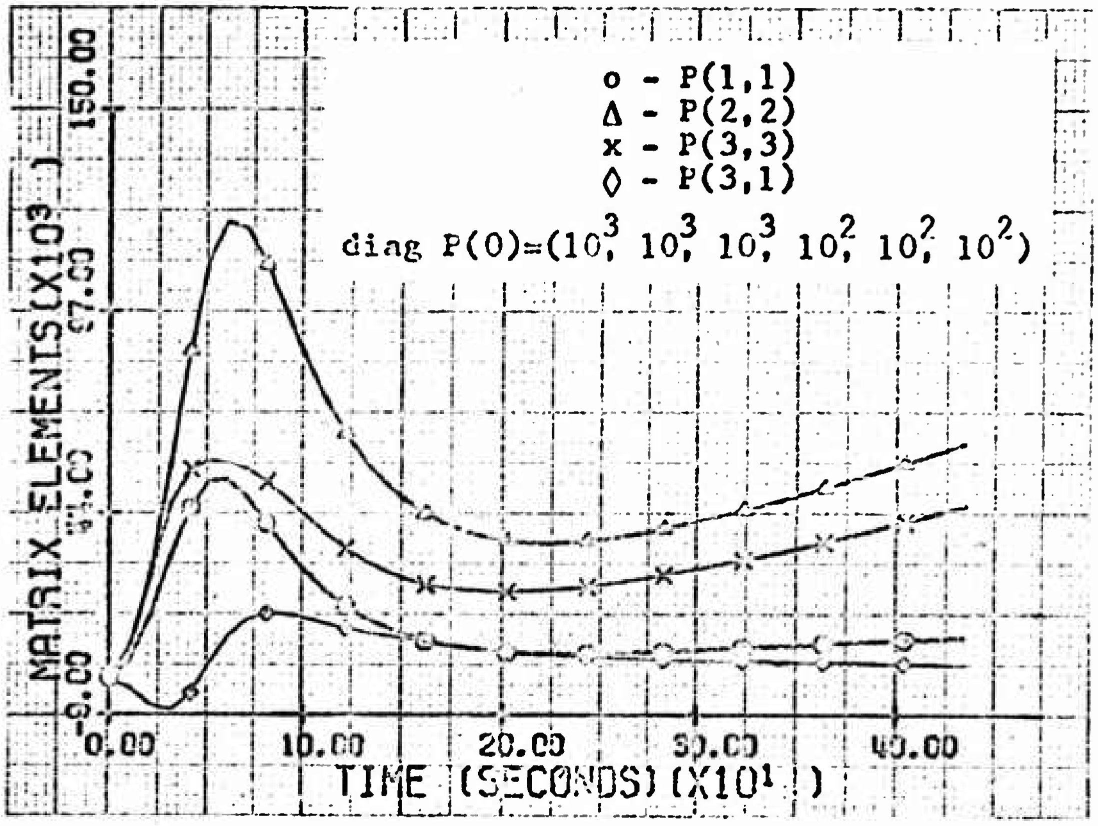

Fig. 31 Satellite 3823 Station 348 Pass #8

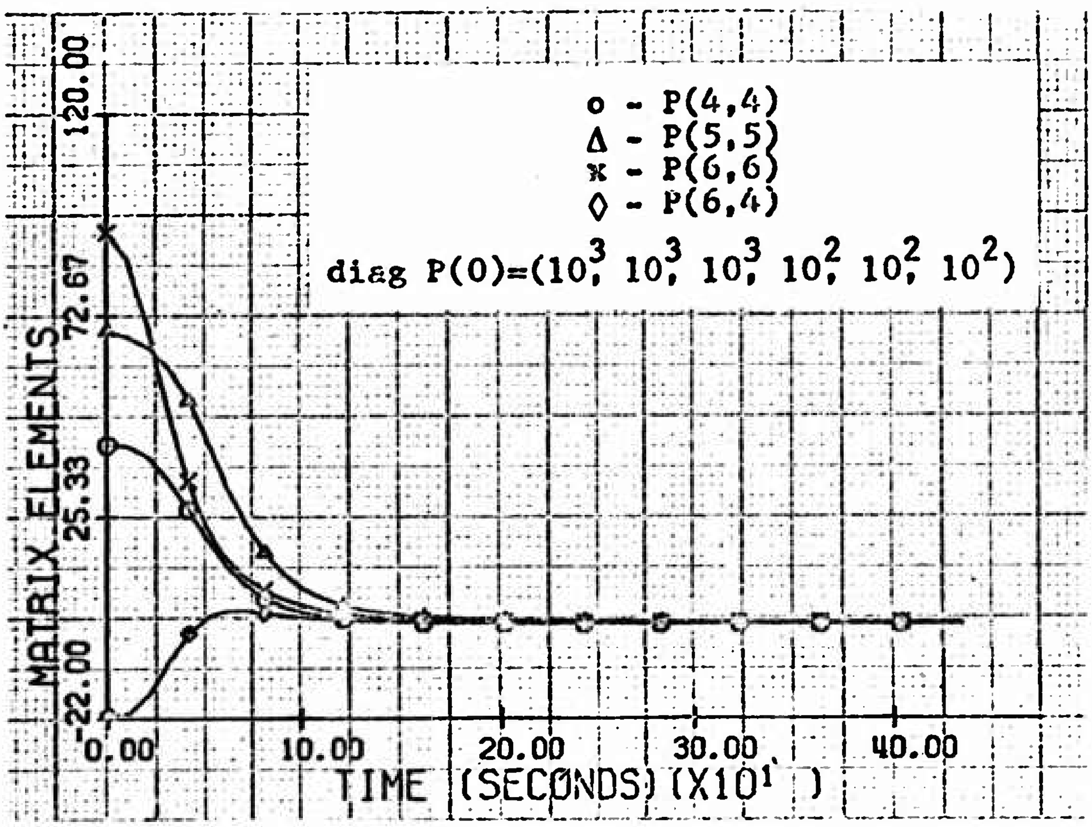

Fig. 32 Satellite 3823 Station 348 Pass #8

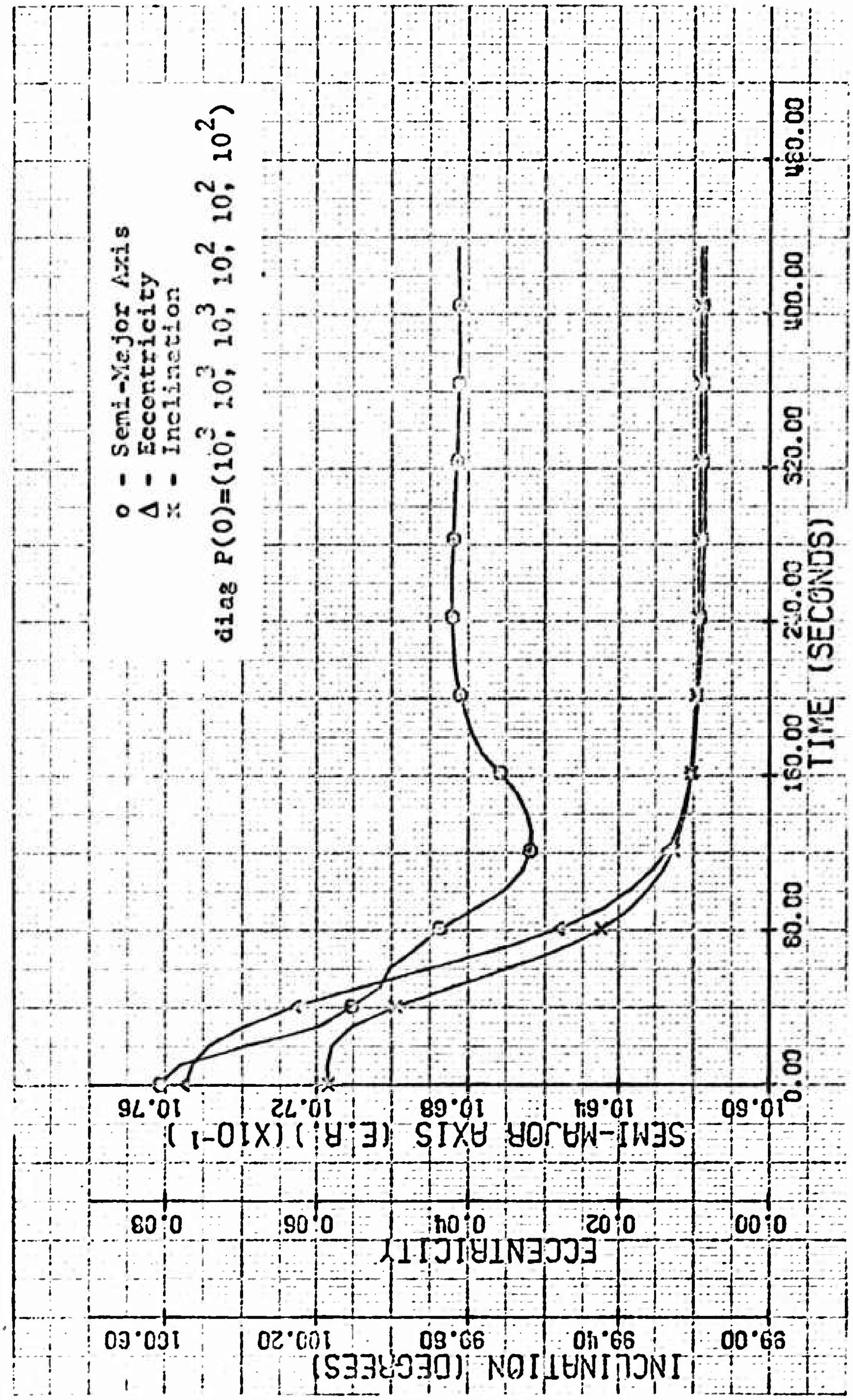

Fig. 33 Satellite 3823 Station 348 Pass #8

Group II Results

The purpose of the six computer runs in this group is to determine if the orbital elements obtained in Group I are repeatable. The passes of each satellite for these correlation runs are chosen as close in time as possible to those in the original runs. Table V presents the satellites and stations used, the number of observations for each run, and the initial, maximum, and minimum values of range and elevation encountered during the radar track. Table VI compares the results obtained in the correlation runs with the corresponding runs from Group I. Also included are the results from an additional correlation run for Satellite 3825. A plot of the orbital elements versus time for each run are shown in Figs. 34-39, while additional curves for the error estimates and error covariance elements are shown in Appendix E.

Group II Analysis

The orbital elements obtained in the correlation runs compare quite favorably with those from Group I. The most noticeable discrepancy is once again the semi-major axis for Satellite 3825. Runs 12 and 16, which use tracking data from Station 349, result in a semi-major axis slightly higher than the SPADATS value. However, Run 15, which uses observations of Satellite 3825

Table V

Group II Computer Runs

Satellite/ Station	Run	diag P(0)	Observations Used	Range (KM)			Elevation (Deg)		
				Init	Min	Max	Init	Min	Max
3826/ 348	11	$(10^4, 10^4, 10^4, 10^2, 10^2, 10^2)$	29 Obser. 10.1 sec. apart	1046	550	1418	8	2	21
3825/ 349	12	$(10^5, 10^5, 10^5, 10^3, 10^3, 10^3)$	27 Obser. 10.1 sec. apart	3936	3272	3936	66	65	83
3823/ 345	13	$(10^3, 10^3, 10^3, 10^3, 10^3, 10^3)$	50 Obser. 6.0 sec. apart	816	772	2127	31	3	33
3824/ 349	14	$(10^5, 10^5, 10^5, 10^3, 10^3, 10^3)$	37 Obser. 10.1 sec. apart	2282	1895	2380	3	2	8
3825/ 345	15	$(10^4, 10^4, 10^4, 10^3, 10^3, 10^3)$	42 Obser. 6.0 sec. apart	1891	1344	1474	35	31	43
3825/ 348	16	$(10^5, 10^5, 10^5, 10^3, 10^3, 10^3)$	27 Obser. 10.1 sec. apart	3616	2661	3616	52	52	77

Table VI

Group II Results

Satellite	Source of Results	Approximate Time of Computation			Semi-Major Axis (ER)	Eccentricity	Inclination (Deg)	Line of Nodes (Deg)
		Pass	Day	Hour				
3826	SPADATS		78	7	1.0445	0.0159	99.051	343.76
3826	Run 5	2	78	10	1.0448	0.0150	99.040	344.30
3826	Run 11	1	78	9	1.0450	0.0155	99.051	344.30
3825	SPADATS		78	3	1.488	0.2789	104.815	342.70
3825	Run 3	12	78	21	1.497	0.2786	104.750	343.17
3825	Run 12	11	78	19	1.492	0.2782	104.680	343.03
3825	Run 15	8	78	11	1.487	0.2796	104.542	343.22
3825	Run 16	8	78	11	1.492	0.2772	104.760	342.89
3824	SPADATS		77	12	1.082	0.0076	98.879	343.24
3824	Run 8	1	77	9	1.083	0.0079	98.890	342.99
3824	Run 14	3	77	14	1.082	0.0071	98.909	343.25
3823	SPADATS		78	3	1.068	0.0050	99.186	344.00
3823	Run 10	8	78	10	1.067	0.0070	99.189	344.34
3823	Run 13	9	78	12	1.069	0.0057	99.046	344.59

from Station 345, yields a semi-major axis which is very close to the SPADATS value. An analysis of Station 345 shows that the measurements for this station have much smaller bias errors and sigmas than Station 349 (Table I). This fact provides a possible explanation for the discrepancy in the answers for the semi-major axis when using tracking data from Station 349.

The results shown in Table VI for Run 12 are taken after 21 observations have been processed. After this point, the orbital elements begin to change and the final answers are quite a bit different from those provided by SPADATS. This change can probably be attributed to the elevation angle which rose quite rapidly near the end, and reached a final value of nearly 83 degrees.

It is possible to obtain steady values for the orbital elements in only 50 seconds (6 observations) for Run 14, and in 48 seconds (8 observations) for Run 13. Very good orbital elements are obtained for Satellite 3824 using tracking data from Station 349, as shown in Fig. 38. Therefore, this station provides only fair elements for an elliptical orbit, but very good elements for a circular orbit.

Group III Results

The actual series of measurements obtained during a single radar track of a satellite often contain from one to six missing observations. The purpose of Group

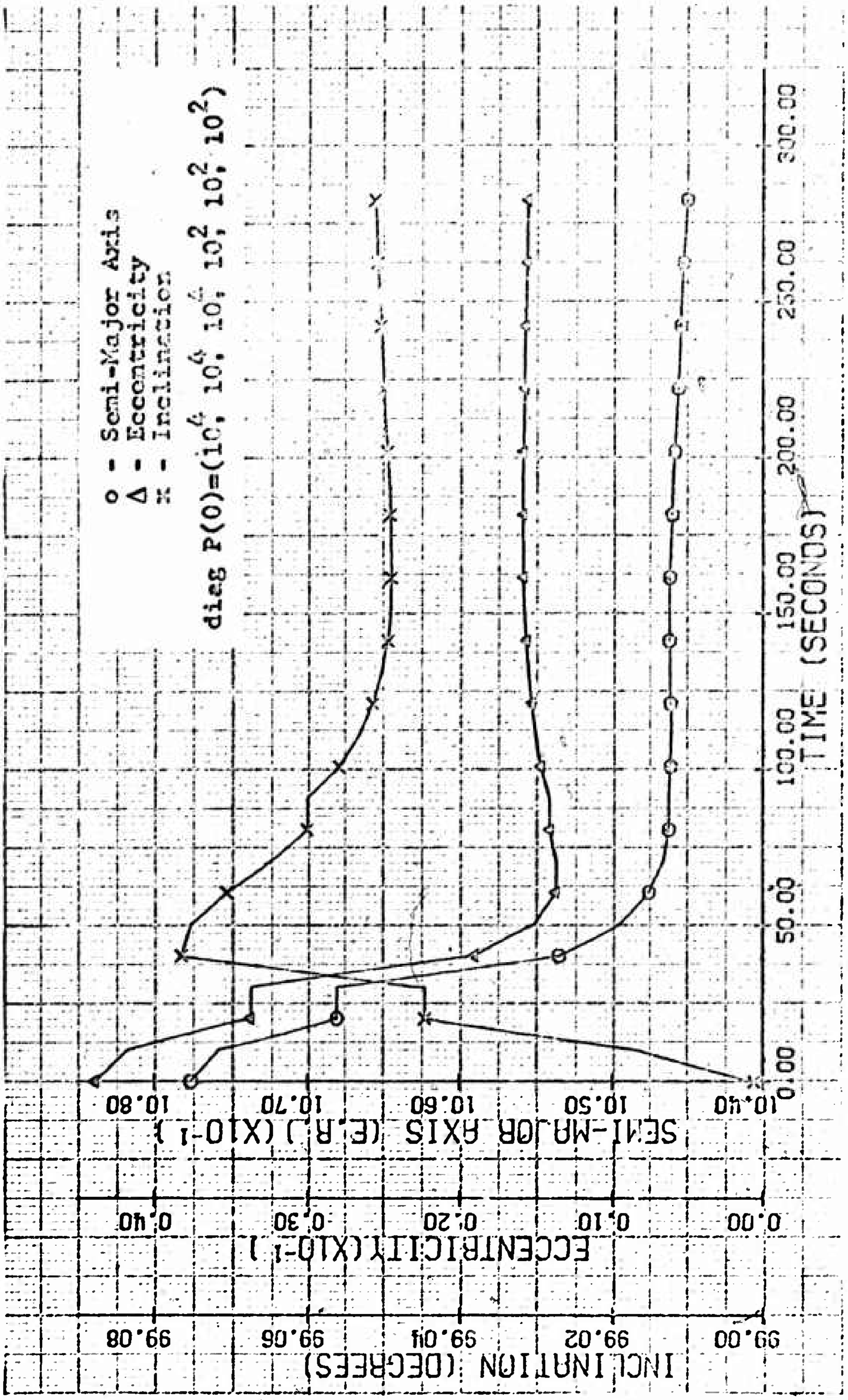

Fig. 34 Satellite 3826 Station 348 Pass #1

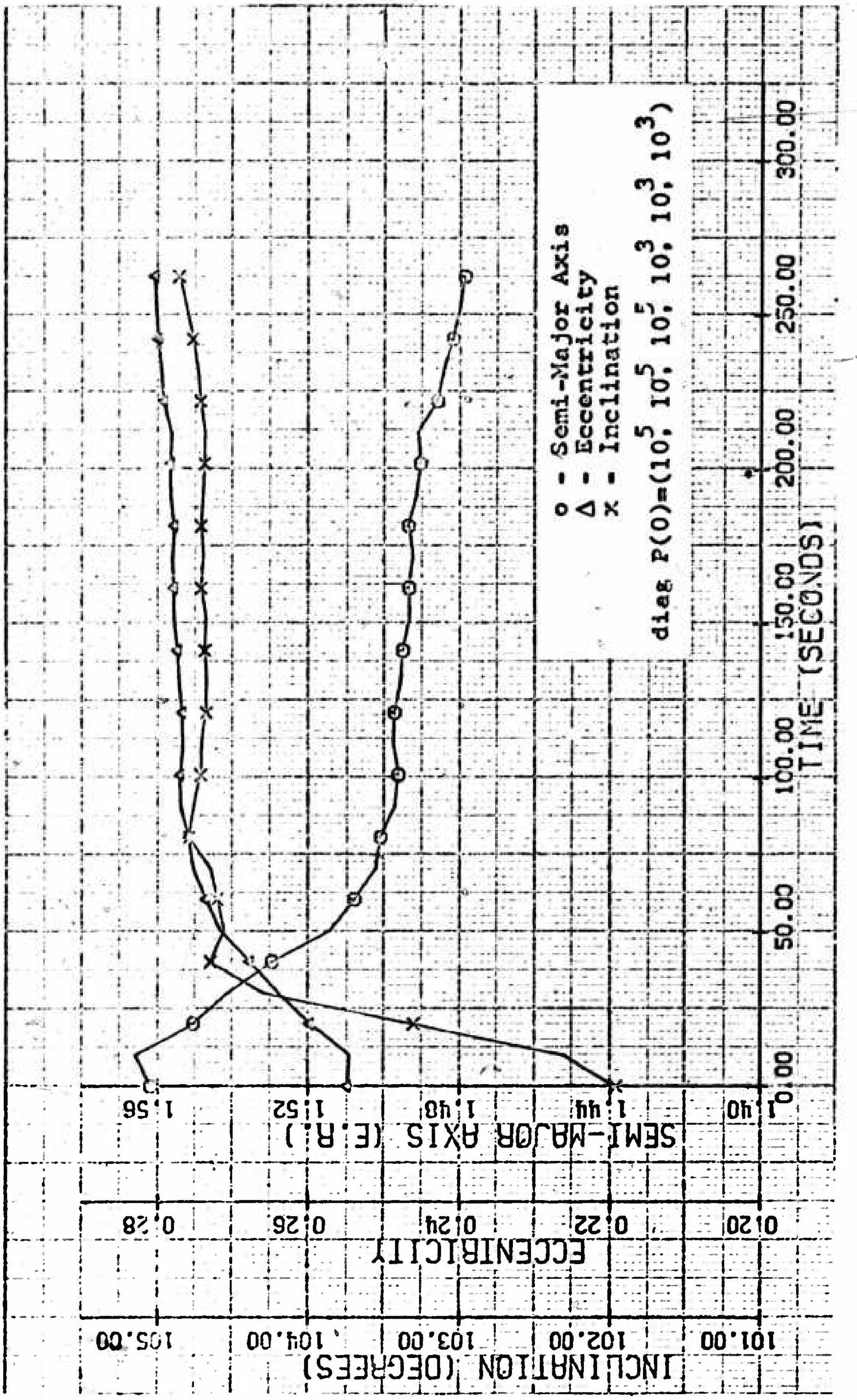

Fig. 35 Satellite 3825 Station 349 Pass #11

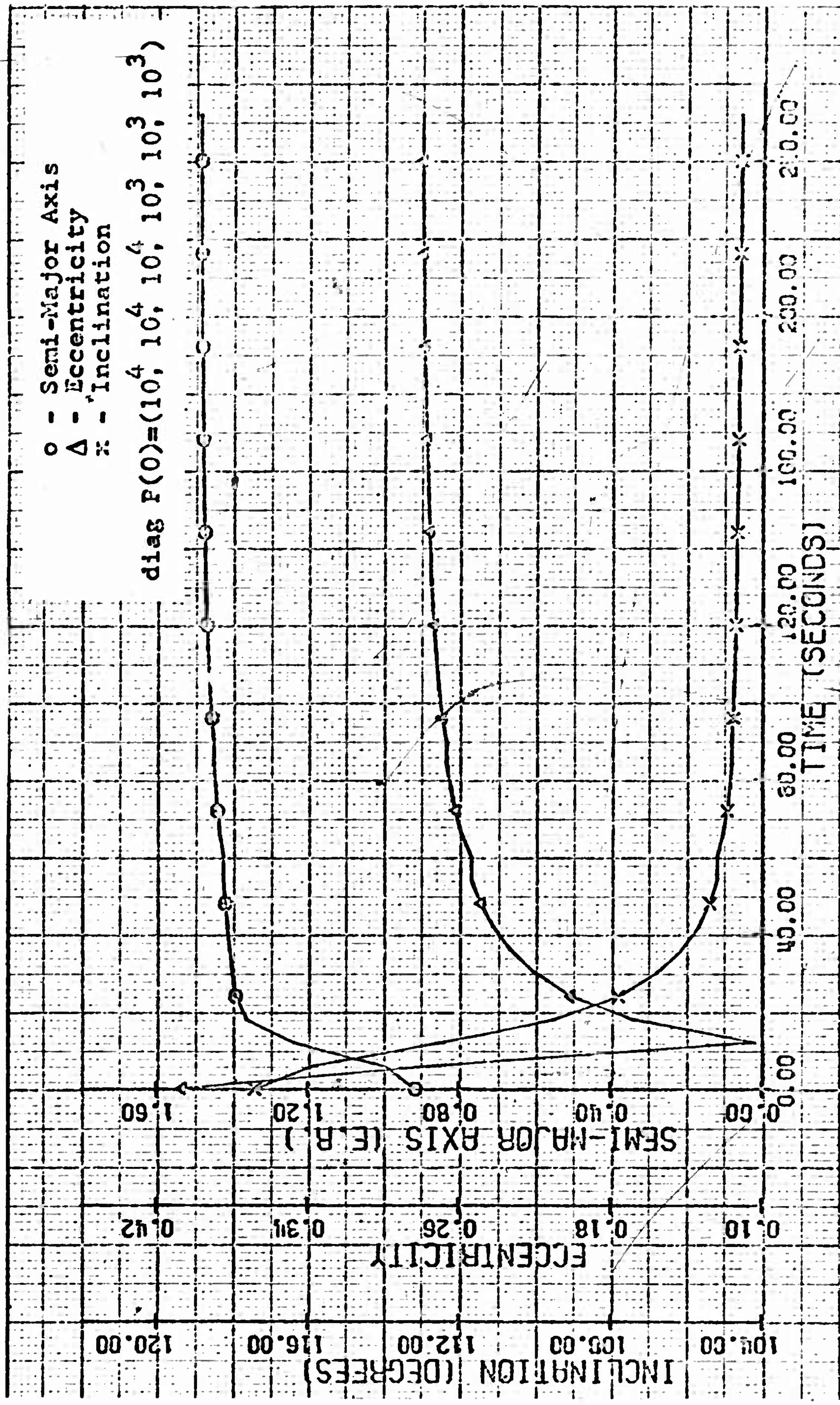

Fig. 36 Satellite 3825 Station 345 Pass #8

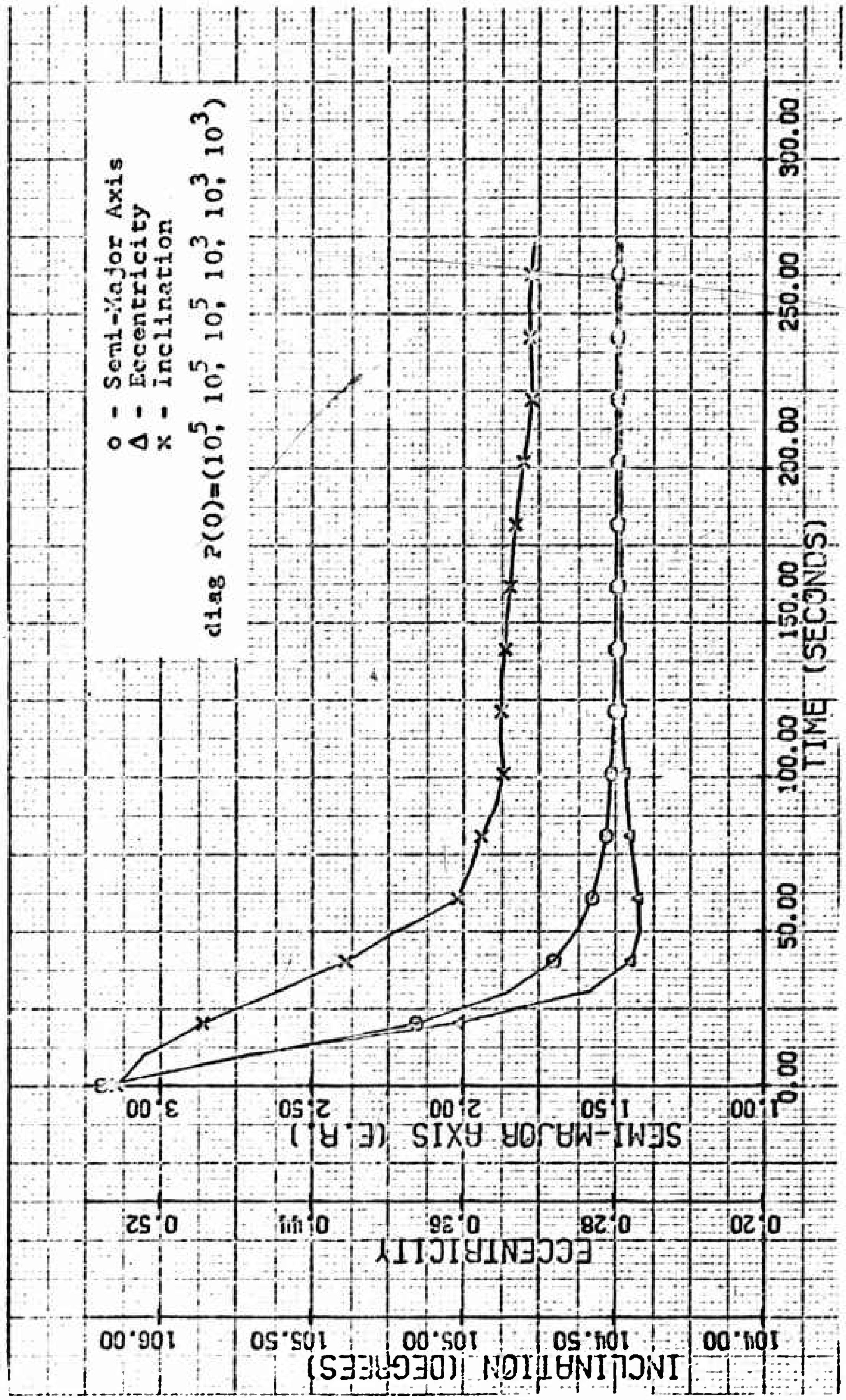

Fig.37 Satellite 3825 Station 348 Pass #8

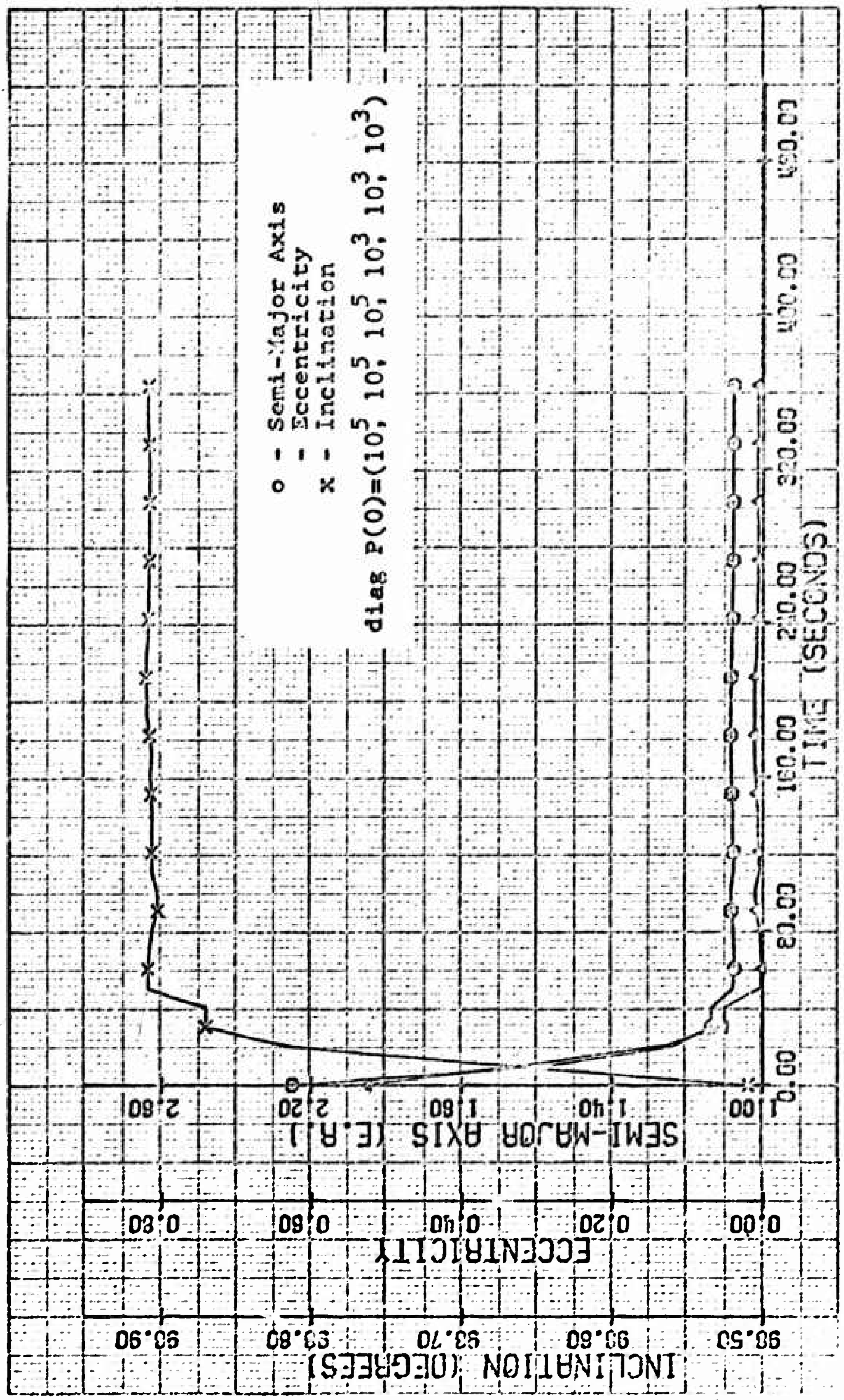

Fig. 38 Satellite 3824 Station 349 Pass #3

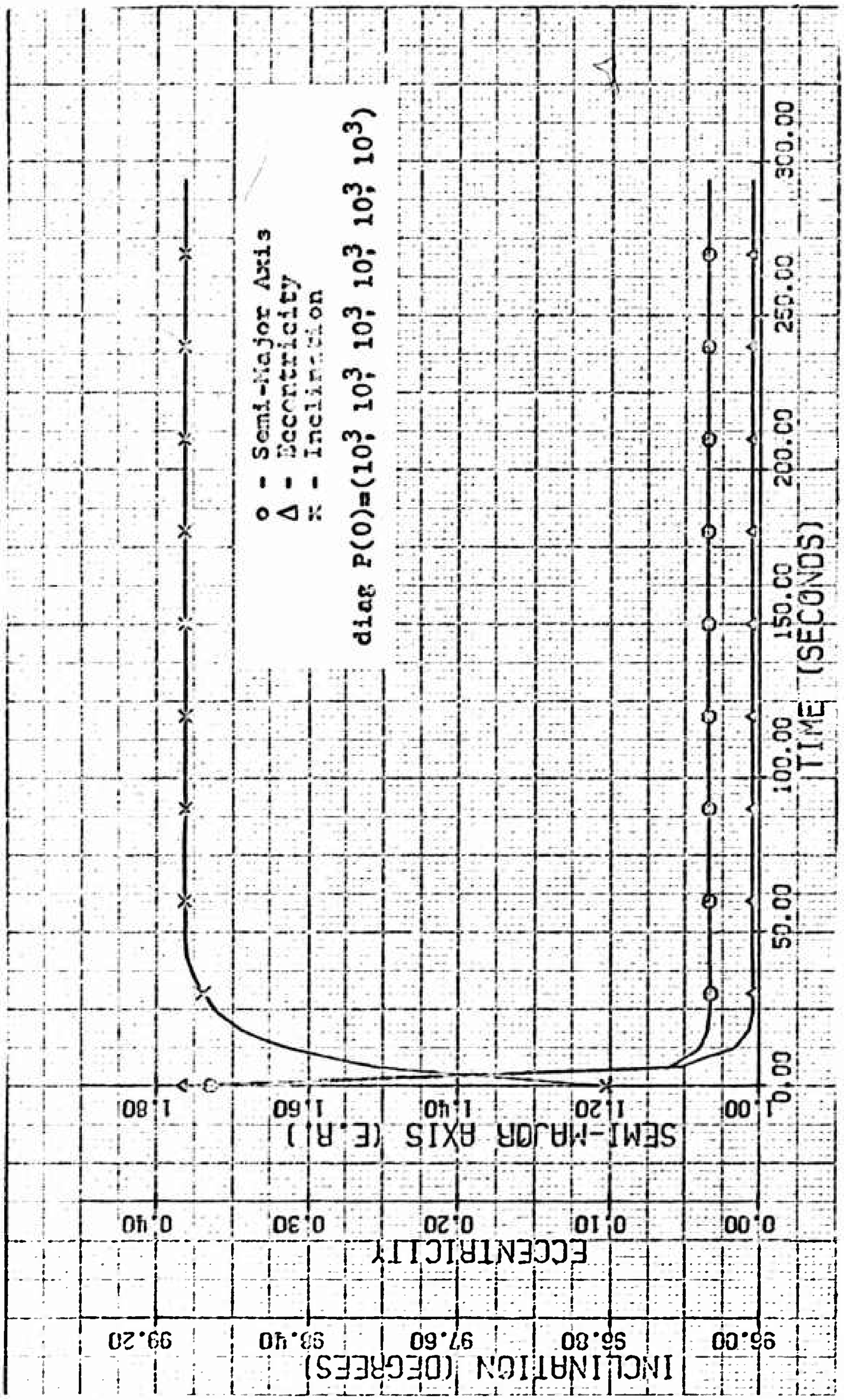

Fig. 39 Satellite 3823 Station 345 Pass #9

III is to determine if a number of consecutive observations can be omitted, and still achieve good results for the orbital elements.

In order to have a basis for comparison, the same tracking data and input P(0) used in Run 2 and Run 7 of Group I are used in this group of runs. These measurements are for Satellites 3825 and 3824, respectively, which represent both highly elliptical and nearly circular orbits. Run 17 uses the data from Run 2 (Satellite 3825) with eight consecutive observations removed after 10 seconds, and Run 18 uses the data from Run 7 (Satellite 3824) with eight consecutive observations removed after 50 seconds. Table VII presents a comparison of the results from the original two runs, the values provided by SPADATS, and the results from Runs 17 and 18. The graphical results are shown in Figs. 40 to 51.

Analysis of Group III Results

The computer algorithm is able to perform quite successfully with eight consecutive missing observations. Preliminary runs (not included) using the same data, but with skips of two, four, and six observations, produced results which are as good or better than the results for Runs 17 and 18.

It is quite interesting to note the behavior of the error covariance matrix, P(-), which is computed

Table VII

Comparison of Results for Group III

Satellite/ Station	Source of Results	Approximate Time of Computation			Semi-Major Axis (ER)	Eccentricity	Inclination (Deg)	Line of Nodes (Deg)
		Pass	Day	Hour				
3824	SPADATS		77	12	1.0822	0.0076	98.879	343.24
3824/ 348	Run 7	1	77	9	1.0803	0.0085	98.897	343.04
3824/ 348	Run 18	1	77	9	1.0805	0.0080	98.895	343.01
3825	SPADATS		78	3	1.4884	0.2789	104.815	342.70
3825/ 349	Run 2	12	78	21	1.4955	0.2773	104.732	343.10
3825/ 349	Run 17	12	78	21	1.4943	0.2772	104.737	343.11

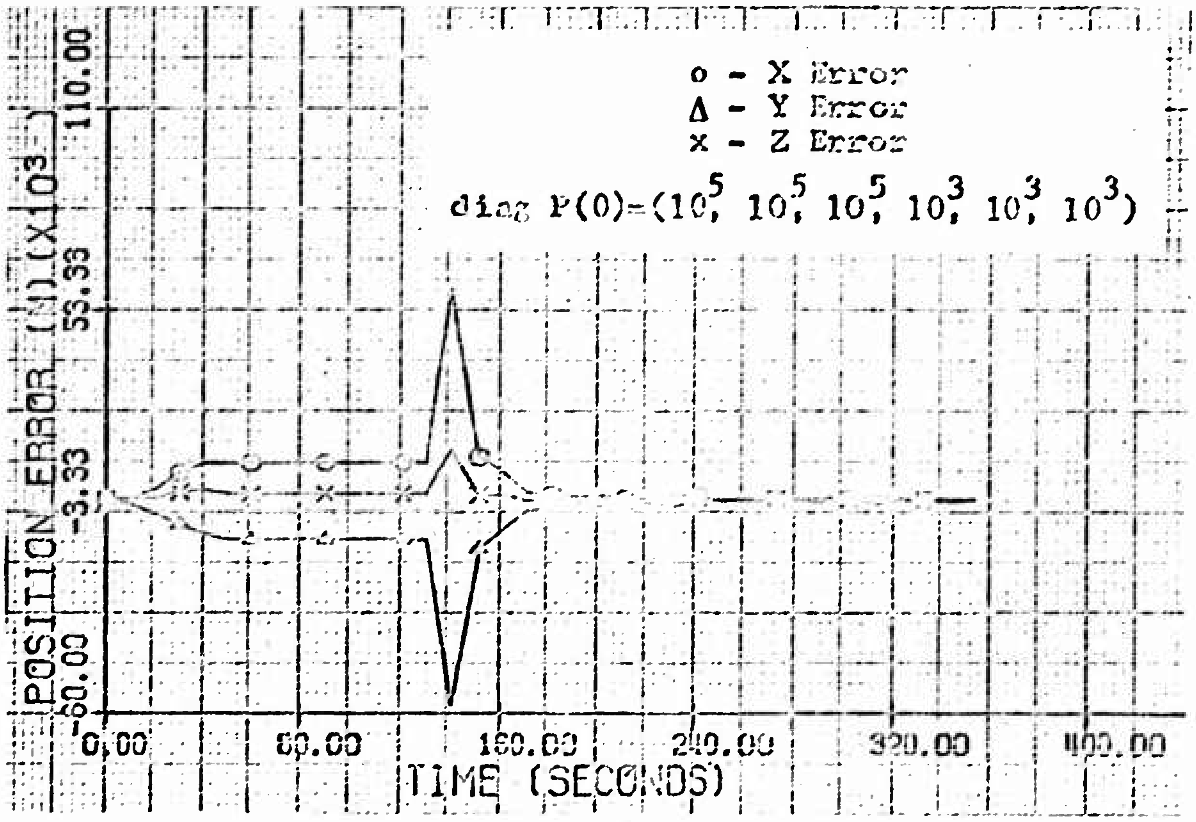

Fig. 40 Satellite 3825 Station 349 Pass #12

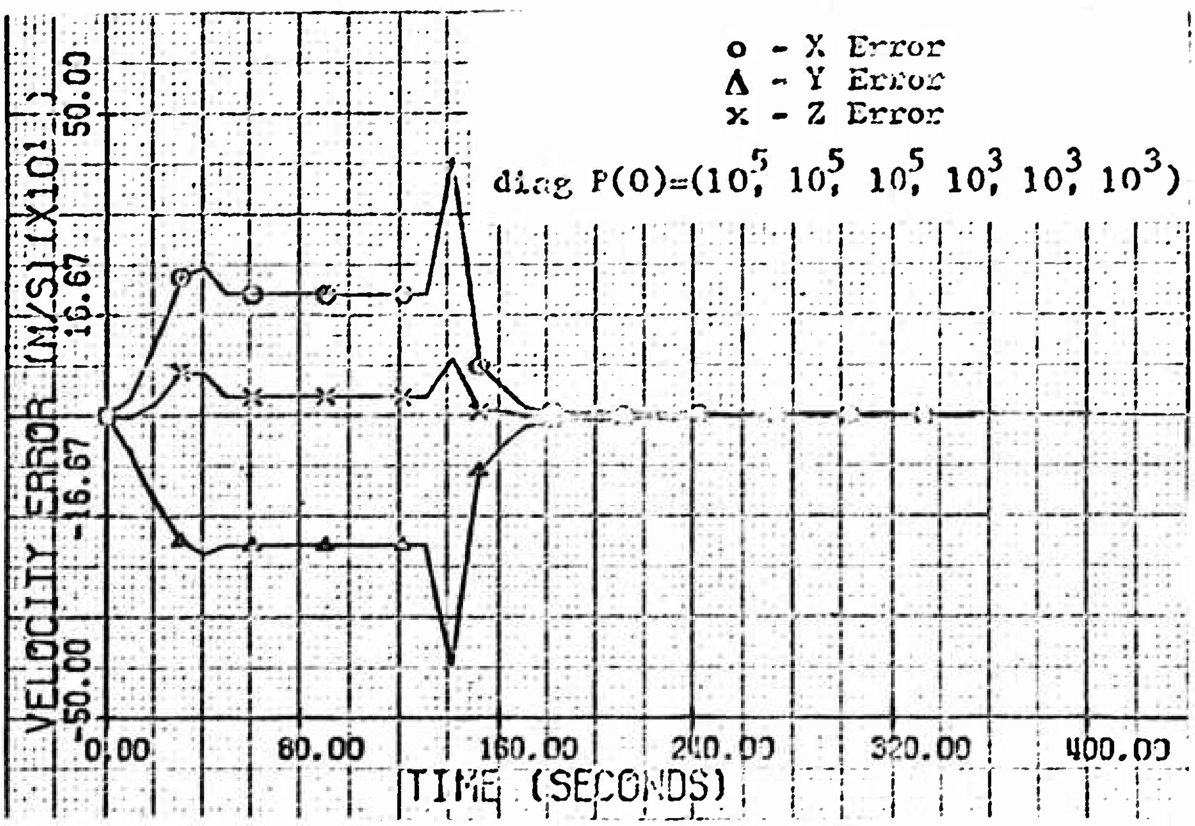

Fig. 41 Satellite 3825 Station 349 Pass #12

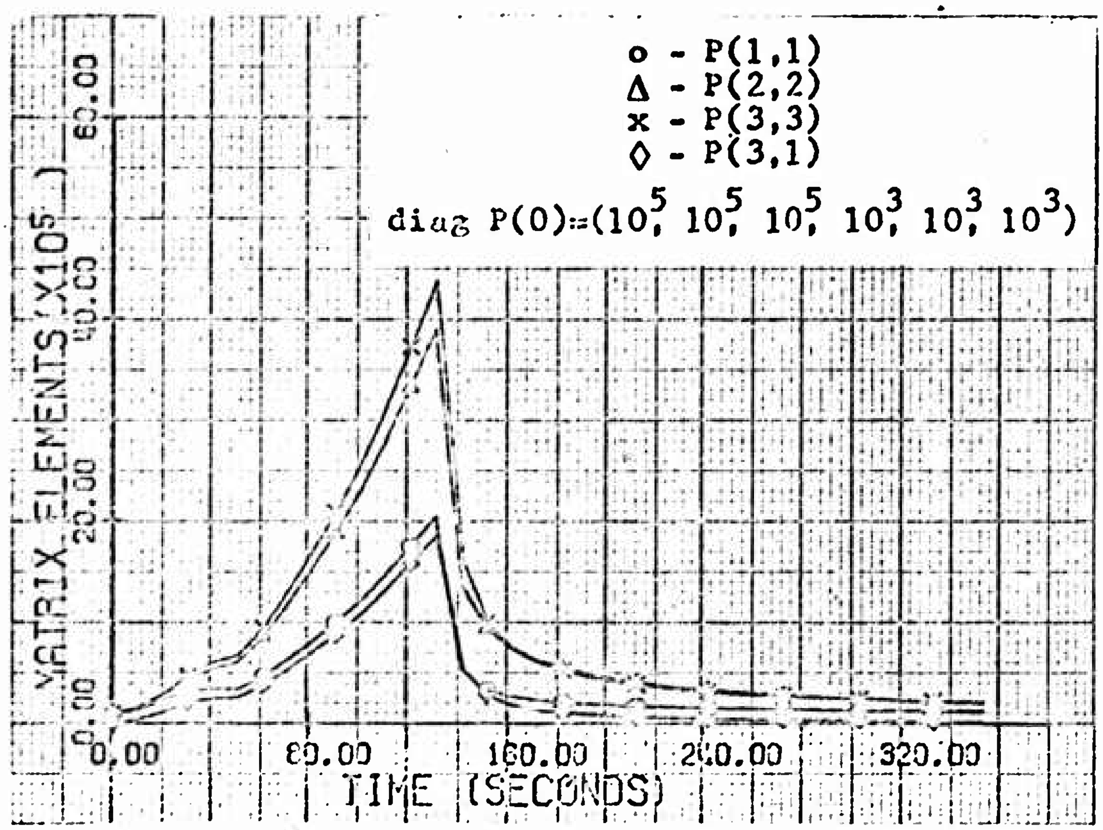

Fig. 42 Satellite 3825 Station 349 Pass #12

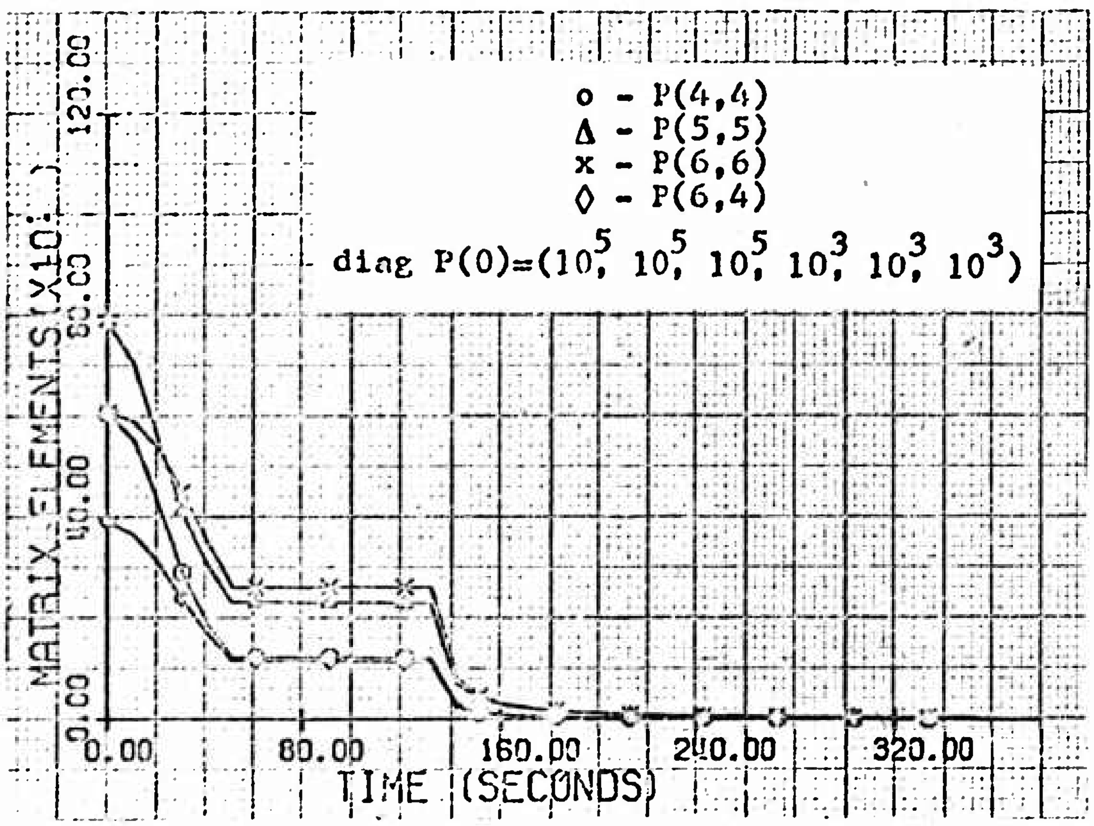

Fig. 43 Satellite 3825 Station 349 Pass #12

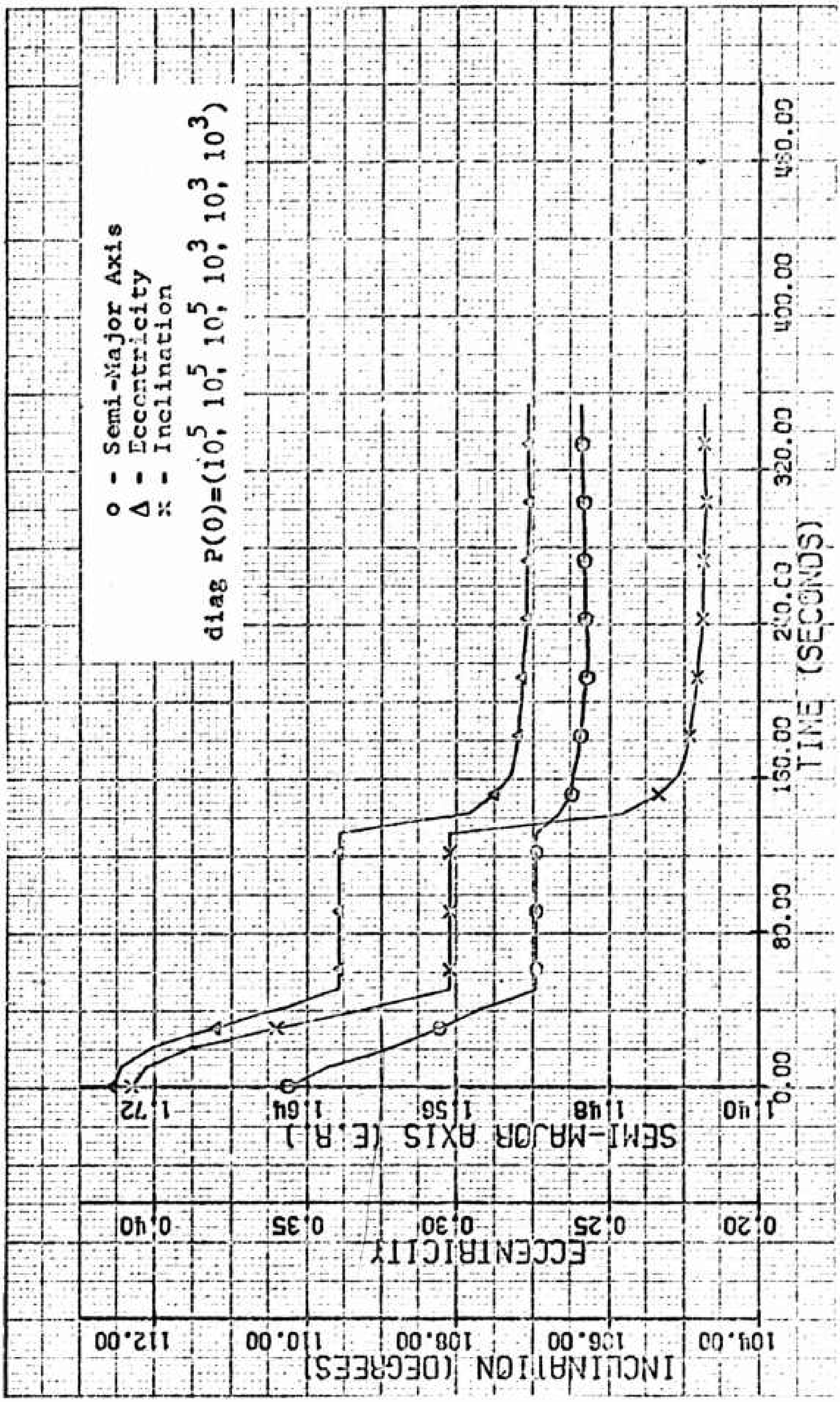

Fig. 44 Satellite 3825 Station 349 Pass #12

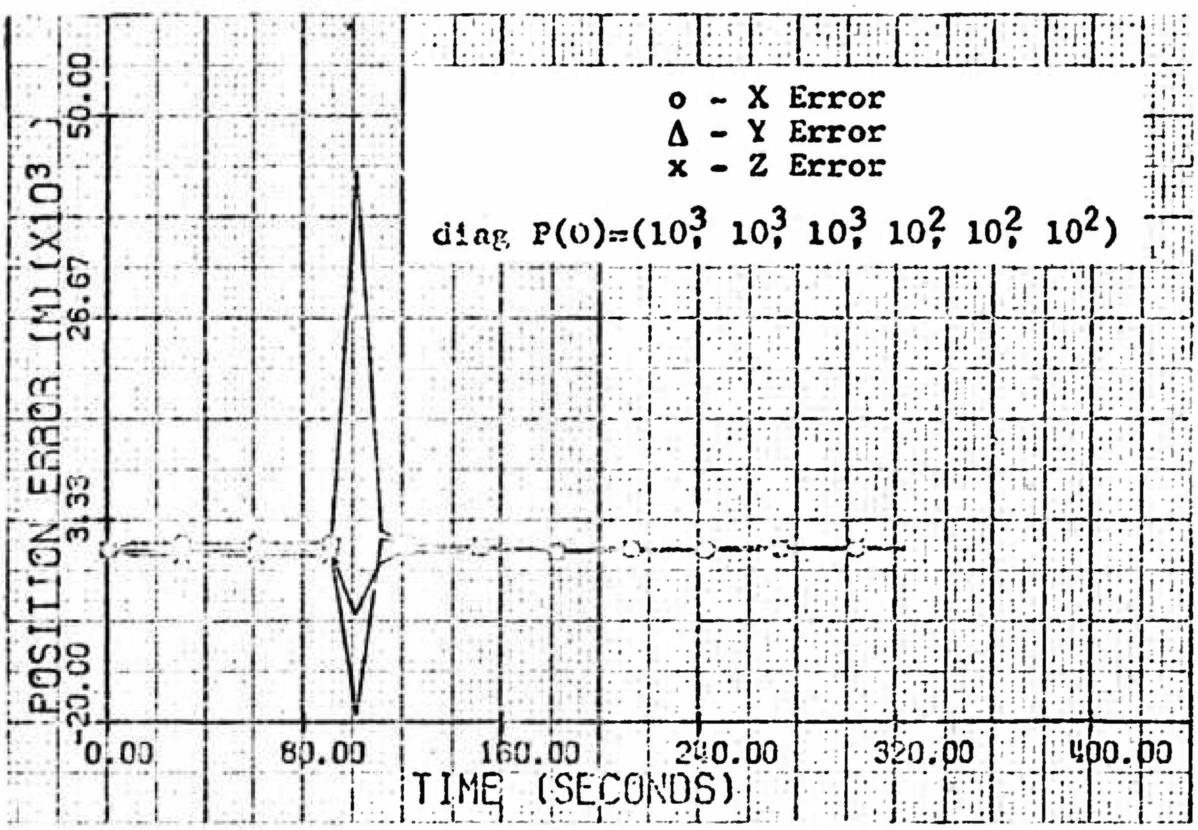

Fig. 45 Satellite 3824 Station 348 Pass #1

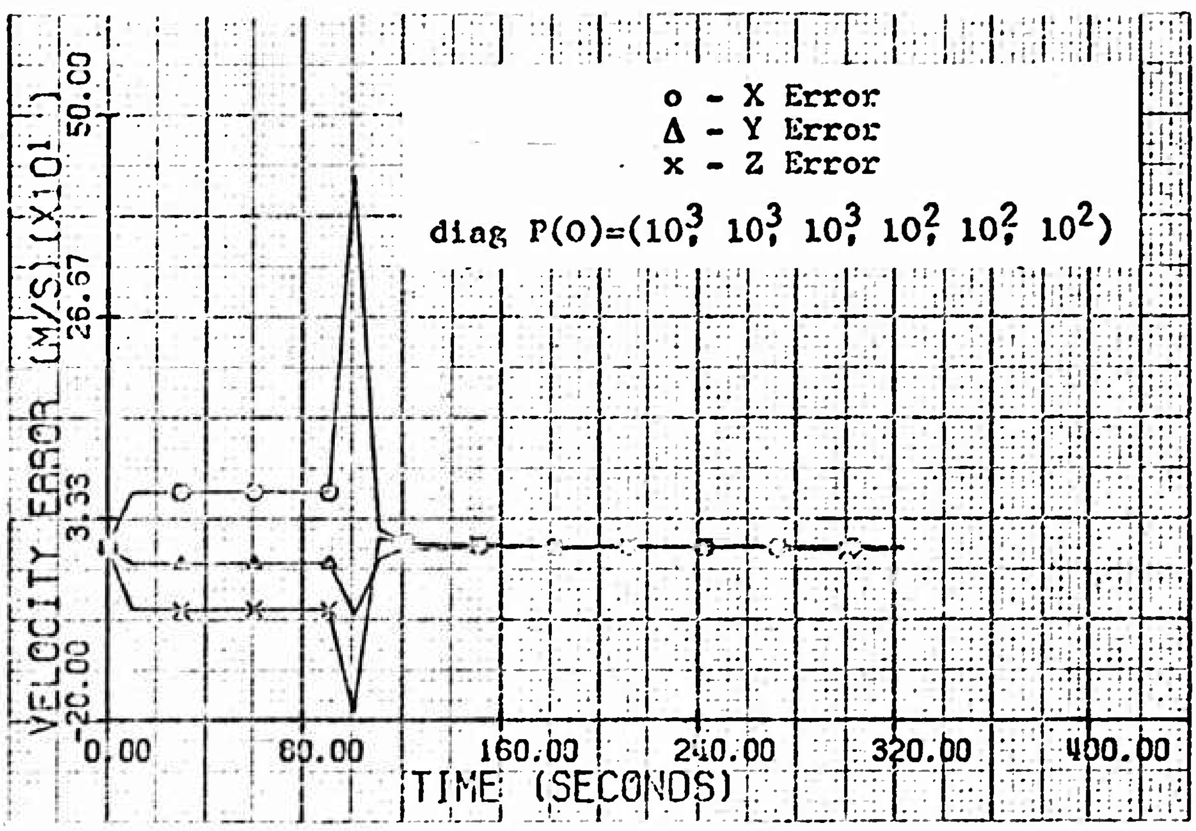

Fig. 46 Satellite 3824 Station 348 Pass #1

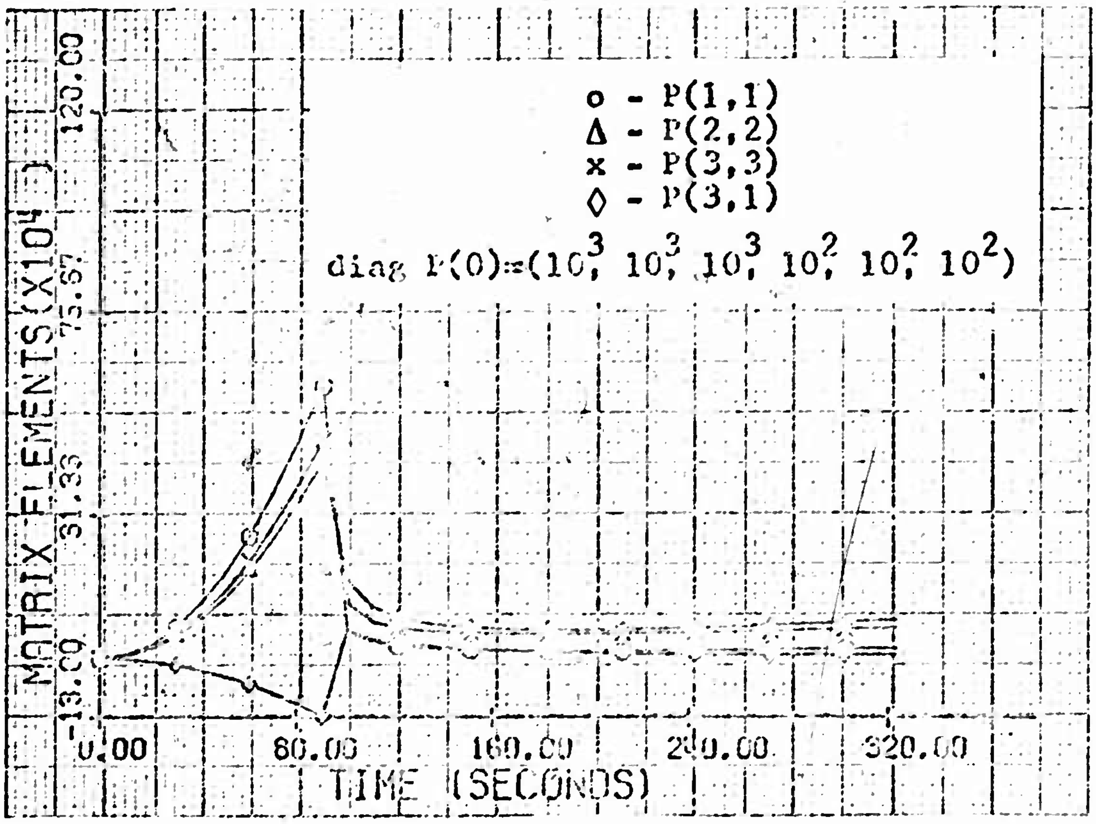

Fig. 47 Satellite 3824 Station 348 Pass #1

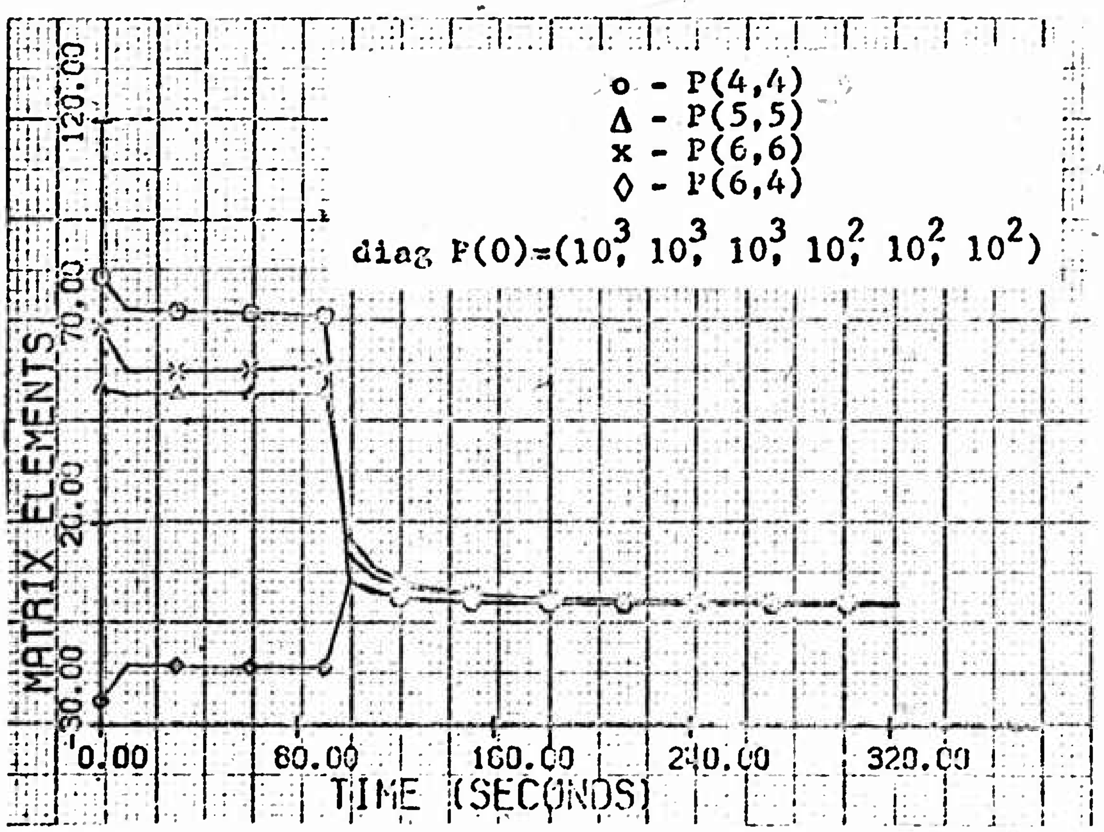

Fig. 48 Satellite 3824 Station 348 Pass #1

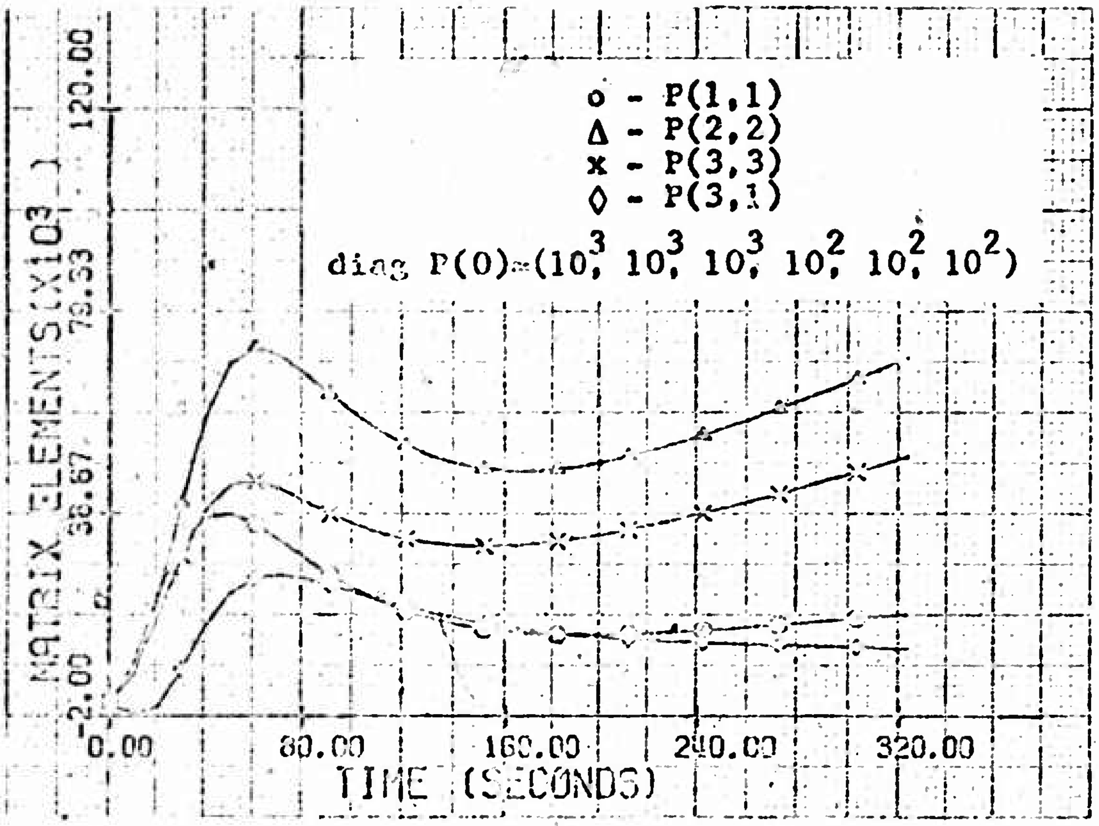

Fig. 49 Satellite 3824 Station 348 Pass #1

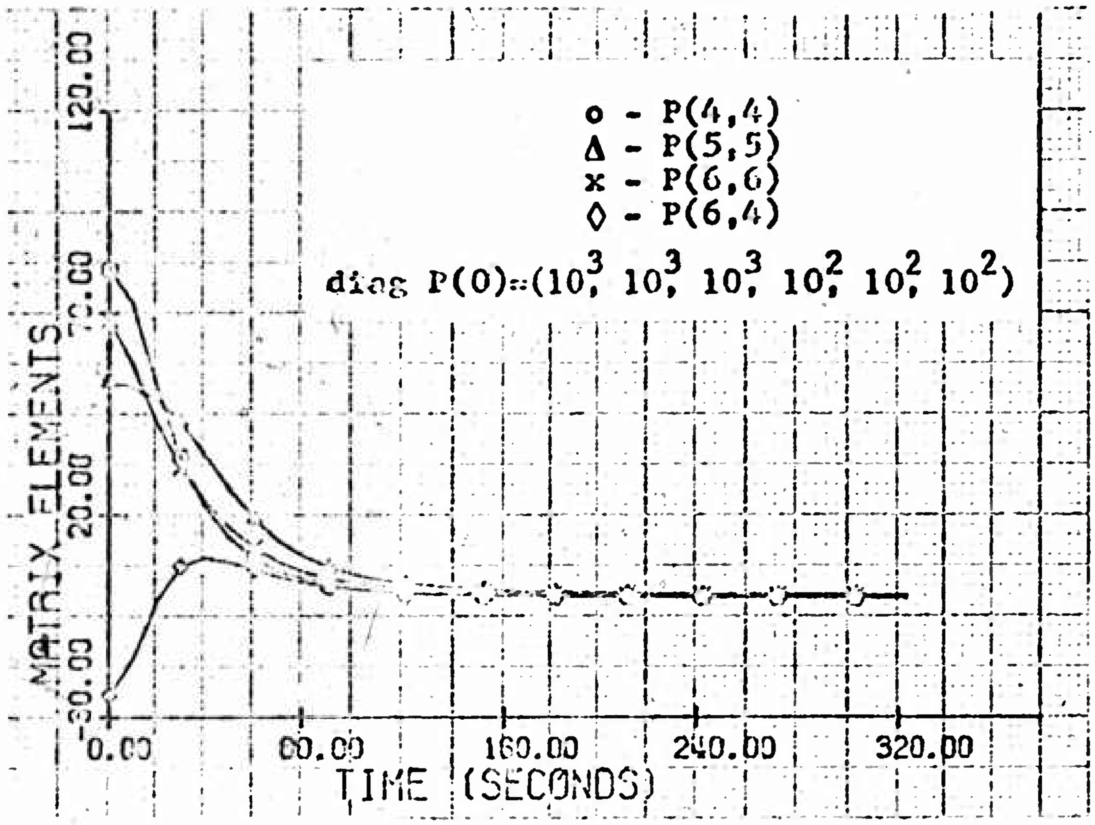

Fig. 50 Satellite 3824 Station 348 Pass #1

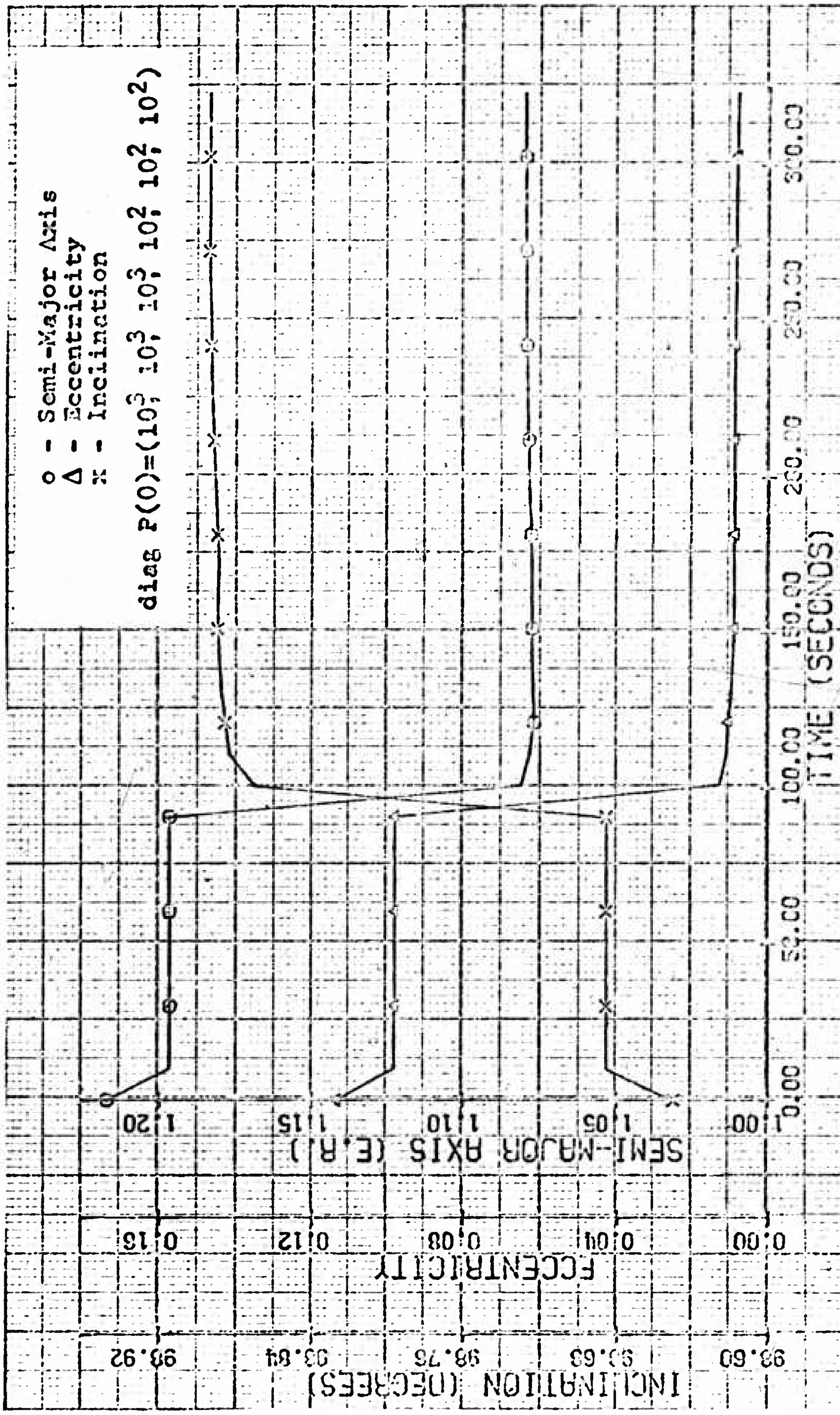

Fig. 51 Satellite 3824 Station 348 Pass #1

from Eq 14 (Chapter II)

$$P_{m+1}(-) = \Phi_m P_m(-)\Phi_m^T \tag{49}$$

This matrix equation is used to propagate the error covariance between measurements. Representative matrix elements are shown in Figs. 49 and 50 for Run 7, which contains no skipped observations, and these can be compared with the same elements in Figs. 47 and 48 for Run 17, which contains eight skipped observations. During the time period covering the skipped data, the matrix elements for the two runs are quite different. However, once the skips are terminated and several new measurements are received, the values for the elements for both runs quickly becomes almost identical. It should be not . that the scales for Figs. 47 and 49 are not the same, causing the curves to appear different, even after the skips are over. But upon closer analysis, it can be seen that after approximately 100 seconds the curves are very similar. Thus, despite the large change in P(-) during the time of the skipped observations, the Kalman Filter continues to operate properly.

Group IV Results

The state transition matrix is computed using Eq (13) which is repeated below

$$\Phi = I + F\Delta t + F^2\Delta t^2/2! + F^3\Delta t^3/3! + \text{h.o.t.} \tag{50}$$

where h.o.t. represent the higher order terms. All the previous runs in Groups I-III used the first four terms shown in Eq (44), and the question arose as to how much the results would be affected by either adding or subtracting terms from this equation. To answer this question, Runs 3, 5, and 9 (Group I) are each repeated using two different equations for the state transition matrix. The initial set of runs use only the first two terms, while the second set of runs include the first five terms.

In each set, the resulting orbital elements are the same as the Group I results out to the fifth or sixth significant digits. A comparison of the graphical results for each set of runs also fails to disclose any appreciable differences. Because of this similarity, the results are not included in this report.

In most of the previous runs, the tracking data from a single radar station during one pass seldom lasts over five minutes. However, one set of data is available from Station 345, Satellite 3825 which covers a time interval of nearly nine minutes. Within this data, there are a number of missing observations (25) which occur in groups of from one to three over the last 270 seconds. Because of the interesting nature of this data, one additional run is made, and the graphical results are shown in Figs. 52-56.

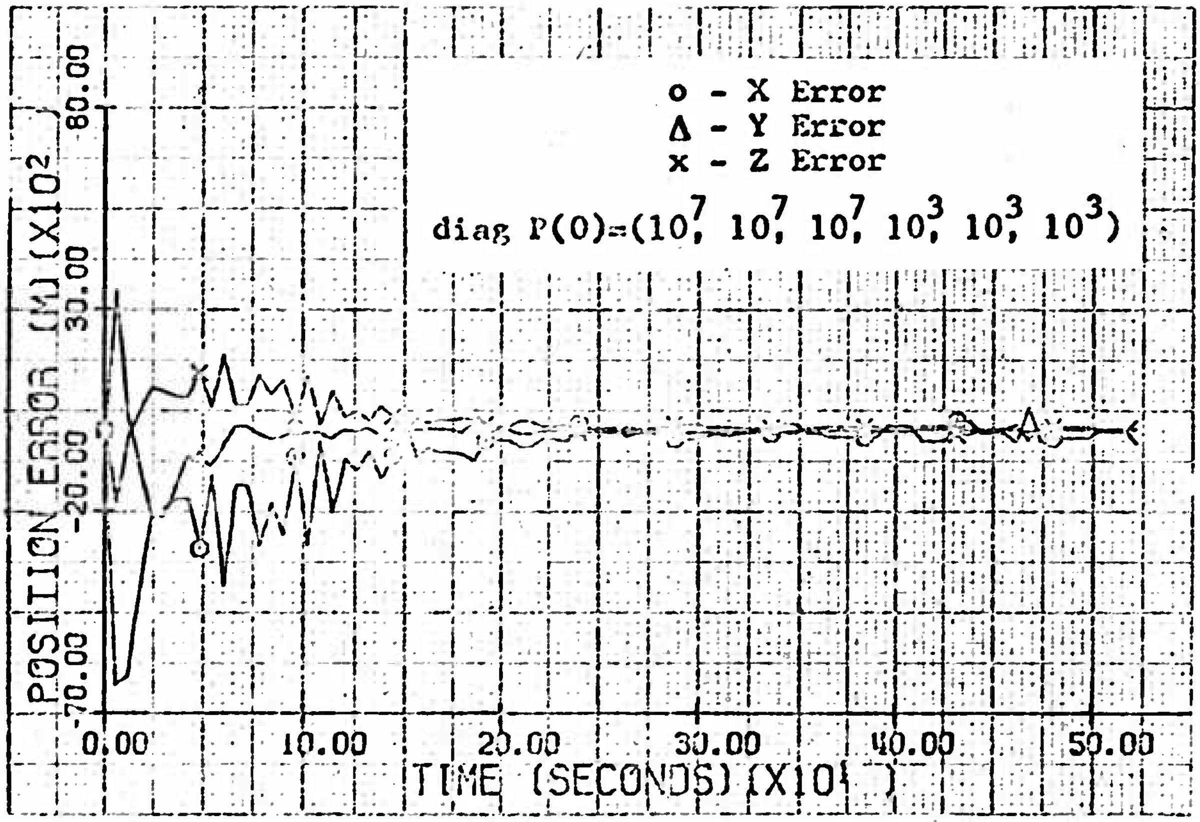

Fig. 52 Satellite 3825 Station 345 Pass #1

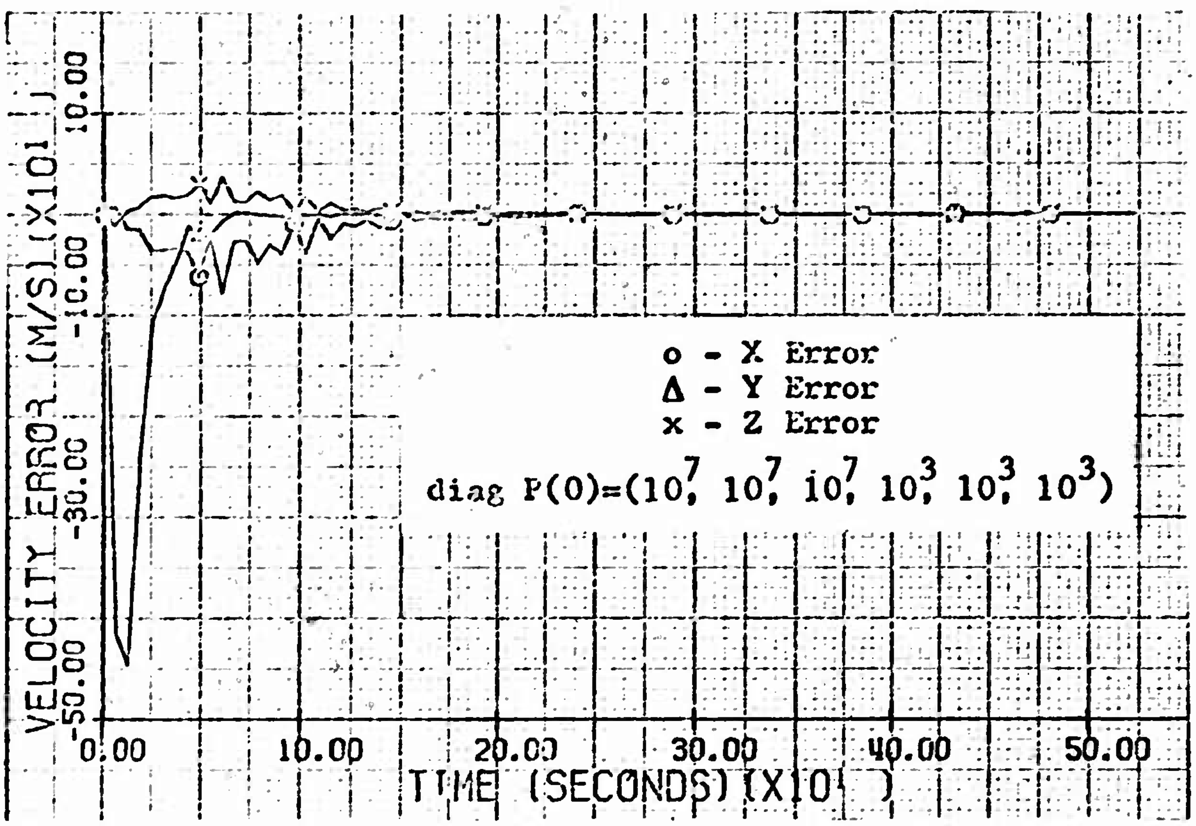

Fig. 53 Satellite 3825 Station 345 Pass #1

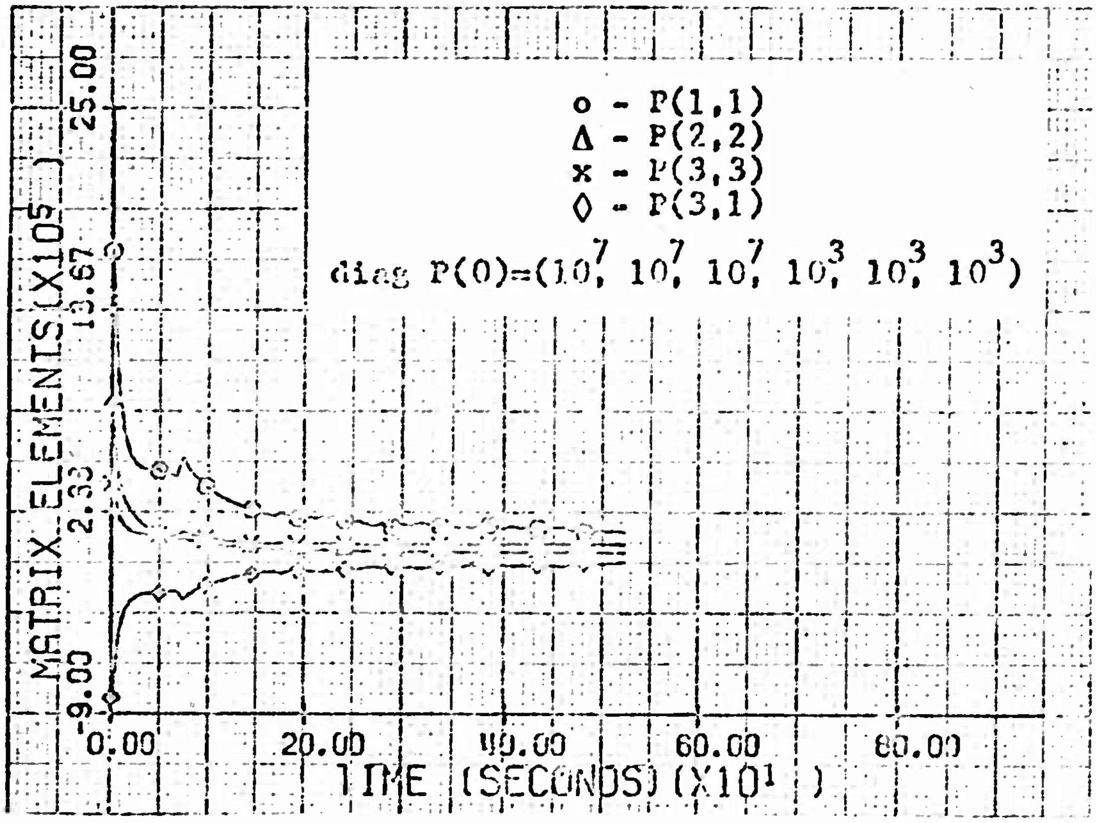

Fig. 54 Satellite 3825 Station 345 Pass #1

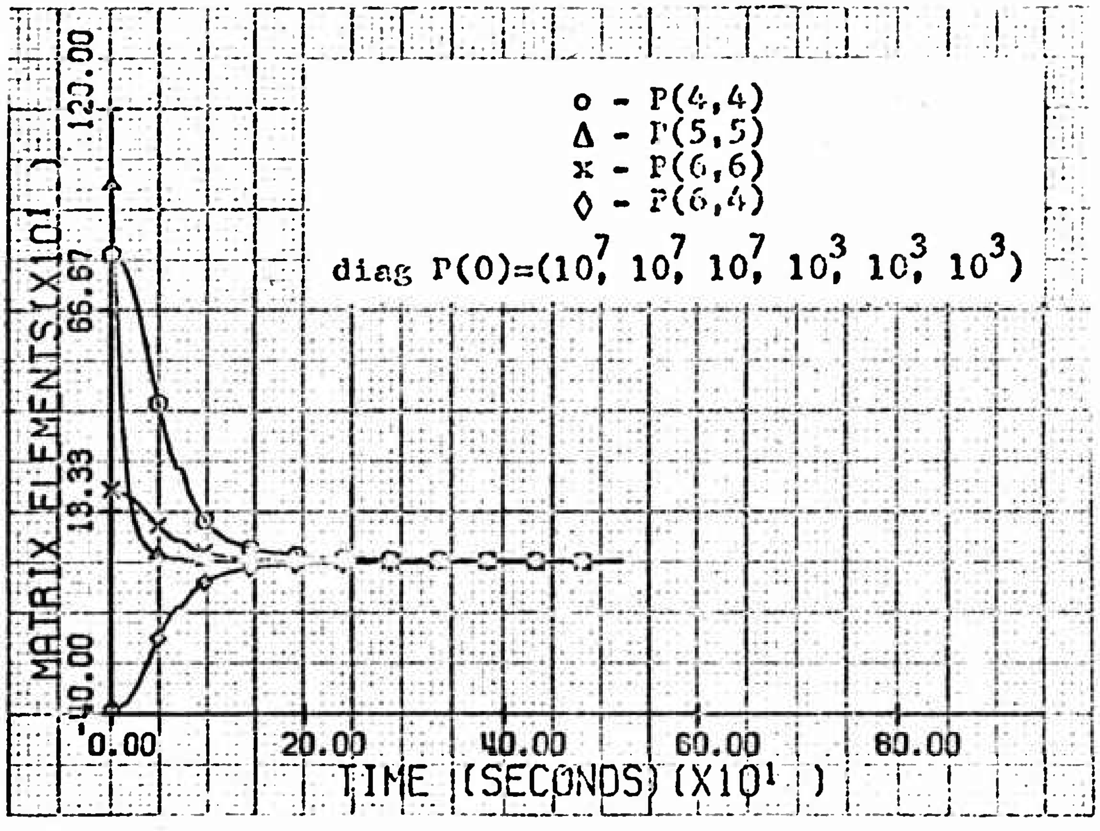

Fig. 55 Satellite 3825 Station 345 Pass #1

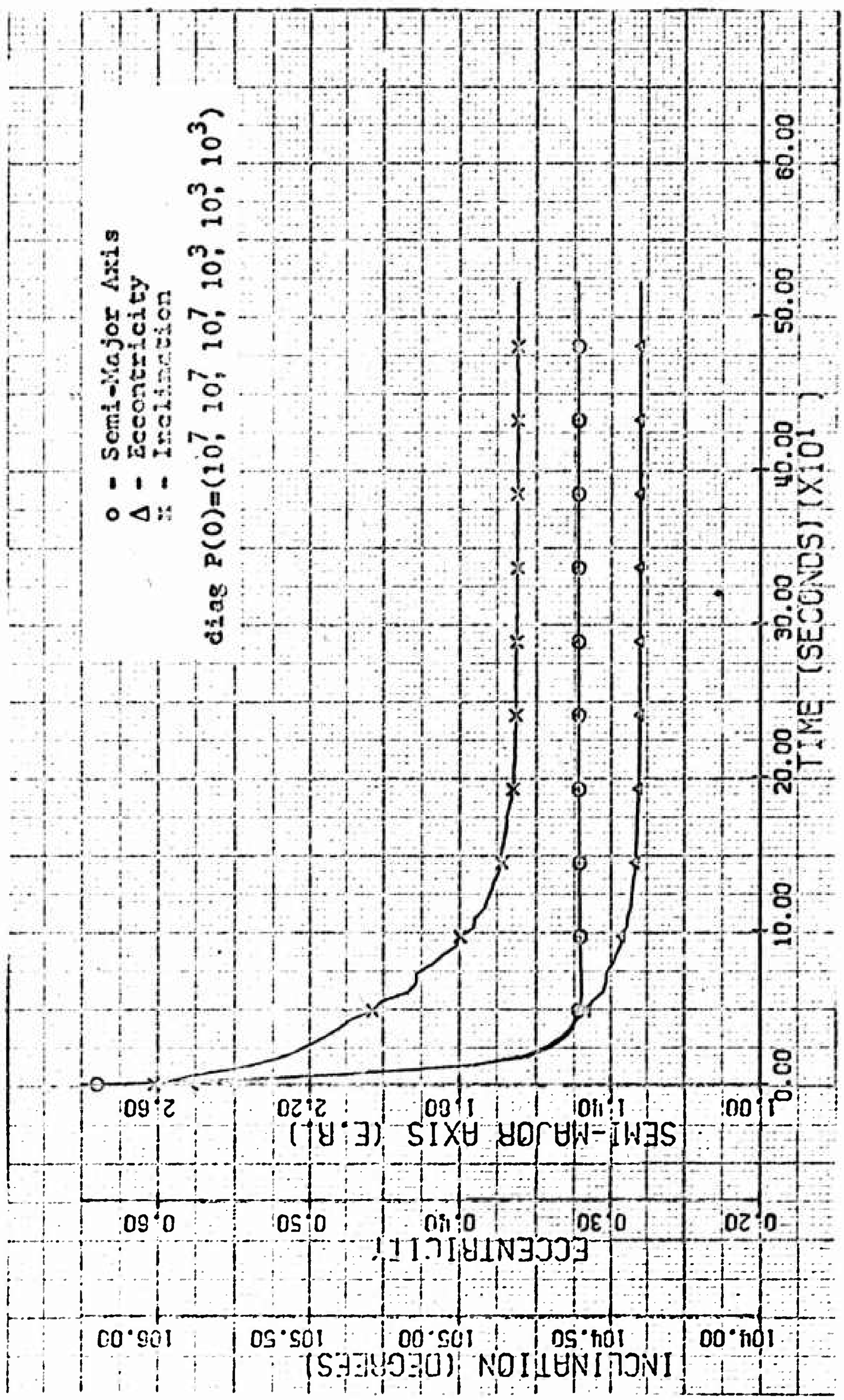

Fig. 56 Satellite 3825 Station 345 Pass #1

Analysis of Group IV Results

In analyzing Eq (50), it is found that the largest elements making up the F matrix are on the order of 0.0001. In addition, the value for Δt is a constant 0.1. Thus, the terms in Eq (50) beyond the second term (i.e. $F^2 \Delta t^2/2 + \cdots$) have maximum magnitudes which are less than 10^{-10}, and these terms are therefore very insignificant when added to the identity matrix I.

The computer run using the nine minute track of Satellite 3825, produces steady values for the orbital elements after approximately 200 seconds, as shown in Fig. 56. By using smaller values for the initial error covariance matrix, P(0), the orbital elements take longer to reach steady levels, once again indicating that it is best to overestimate diag P(0). The run could have been terminated after the steady values are reached since the orbital elements are essentially unchanged for the final 55 observations. The final value for the semi-major axis (1.4927) is higher than the value provided by SPADATS (1.488). Therefore, even by using data from two different tracking stations (345 and 349), a consistently high value has been obtained for the semi-major axis for Satellite 3825. A possible explanation could be that the value computed by SPADATS is in error.

VI. Conclusions and Recommendations

Conclusions

1. The application of the Kalman Filter to determine the orbit of a space vehicle is quite successful. For each of the four satellites used in the study, it is possible to obtain convergence of the error estimates to values less than 0.1 per cent of the actual values of position and velocity. When measurements spaced six seconds apart are used, convergence is achieved with a minimum of eight observations. For measurements ten seconds apart, a minimum of six are needed to obtain convergence. The value chosen for the initial error covariance matrix greatly affects the rate of convergence, and in general, overestimation provides steady orbital elements in the shortest time. Once convergence is reached, the optimal estimate of the trajectory is obtained, and additional measurements fail to improve this trajectory.

2. The orbital elements are repeatable for three of the satellites when using tracking data from different radar stations. The orbital elements are repeatable for the fourth satellite when using tracking data from the same radar station for two consecutive passes.

3. The computer algorithm is capable of integrating for over 80 seconds with no tendency to diverge.

Therefore, as many as eight consecutive measurements spaced 10 seconds apart can be missing from within a track of a satellite, and the program will still provide good results. If the missing observations occur before convergence is reached, then several additional measurements are needed following the skips in order to obtain steady values for the orbital elements.

4. When computing the state transition matrix, only the first two terms of the series expansion are needed, and all additional terms are negligible.

Recommendations

The following topics are suggested as areas for further study.

1. Use tracking data for satellites with low and intermediate inclination angles.

2. Include perturbation terms, such as atmospheric drag, solar drag, and additional zonal harmonic terms, in the nonlinear state equations.

3. Include tracking data for both continuously thrusting and intermittently thrusting vehicles.

4. Extend computer algorithm to include iterative differential correction techniques as outlined by Morrison (Ref 14:428-482).

5. Extend the dimension of the state vector and filter matrices in order to include the station biases and sigmas in the estimation process (Ref 7:241-242).

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Appendix A

Initial Nominal Trajectory Equations

The equations used to compute the initial nominal trajectory are presented in this appendix. The input data needed includes the following:

1. Height of tracking station, h(meters)
2. Latitude of tracking station, ϕ (degrees)
3. Longitude of tracking station, θ (degrees)
4. Slant range of satellite, ρ (meters)
5. Slant range rate of satellite, $\dot{\rho}$ (meters/second)
6. Azimuth of satellite, A(degrees)
7. Elevation of satellite, E(degrees)
8. Azimuth rate of satellite, $\dot{A}$(degrees/second)
9. Elevation rate of satellite, $\dot{E}$(degrees/second)

Computation of Station Coordinates in Rotating Frame

$$\underline{R} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -(C+H)\cos(\theta)\cos(\phi) \\ -(C+H)\cos(\phi)\sin(\theta) \\ -(S+H)\sin(\phi) \end{bmatrix} \tag{A-1}$$

where H is the station height in earth radii, and

$$C = 1/\left[1 - (2f-f^2)\sin^2(\phi)\right]^{1/2} \tag{A-2}$$

$$S = C(1-f)^2 \tag{A-3}$$

Computation of Vectors L and $\dot{L}$ in Rotating Frame

$$\underline{L} = \begin{bmatrix} LX \\ LY \\ LZ \end{bmatrix} = \begin{bmatrix} \cos(\theta)(-L1+L2) - \sin(\theta)(L3) \\ \sin(\theta)(-L1+L2) + \cos(\theta)(L3) \\ \cos(\phi)\cos(h)\cos(A) + \sin(\phi)\sin(h) \end{bmatrix} \quad \text{(A-4)}$$

where

$$L1 = \sin(\phi)\cos(h)\cos(A) \quad \text{(A-5)}$$

$$L2 = \cos(\phi)\sin(h) \quad \text{(A-6)}$$

$$L3 = \cos(h)\sin(A) \quad \text{(A-7)}$$

$$\dot{\underline{L}} = \begin{bmatrix} LXD \\ LYD \\ LZD \end{bmatrix} \quad \text{(A-8)}$$

where

$$\begin{aligned} LXD = {} & (\dot{A})\cos(\theta)\sin(\phi)\sin(A)\cos(h)+ \\ & (\dot{E})\cos(\theta)\sin(\phi)\sin(h)\cos(A)+ \\ & (\dot{E})\cos(\theta)\cos(\phi)\cos(h)- \\ & (\dot{A})\cos(A)\sin(\theta)\cos(h)+ \\ & (\dot{E})\sin(\theta)\sin(A)\sin(h) \end{aligned} \quad \text{(A-9)}$$

$$\begin{aligned} LYD = {} & (\dot{A})\sin(\theta)\sin(\phi)\sin(A)\cos(h)+ \\ & (\dot{E})\sin(\theta)\sin(\phi)\sin(h)\cos(A)+ \\ & (\dot{E})\sin(\theta)\cos(\phi)\cos(h)+ \\ & (\dot{A})\cos(\theta)\cos(A)\cos(h)- \\ & (\dot{E})\cos(\theta)\sin(h)\sin(A) \end{aligned} \quad \text{(A-10)}$$

$$LZD = (-\dot{A})\cos(\phi)\sin(A)\cos(h) - (\dot{E})\cos(\phi)\cos(A)\sin(h) + (\dot{E})\sin(\phi)\cos(h) \tag{A-11}$$

<u>Computation of Satellite Position Components in Rotating Frame</u>

$$\underline{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \rho\underline{L} - \underline{R} \tag{A-12}$$

<u>Computation of Satellite Velocity Components in Rotating Frame</u>

$$\dot{\underline{r}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \rho\dot{\underline{L}} + \dot{\rho}\underline{L} \tag{A-13}$$

Appendix B

System Description Matrix

The system description matrix F is made up of the partial derivatives of the equations of motion with respect to the states x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$, as given by Eq (23) in Chapter III.

$$F_{41} = \mu/r^3\left[3(x/r)^2-1\right]\left[1+H_a\right] - xH_{ax} + \omega^2 \tag{B-1}$$

$$F_{42} = \mu x/r^3\ (3y/r^2)(1+H_a) - H_{ay} \tag{B-2}$$

$$F_{43} = \mu x/r^3\ (3z/r^2)(1+H_a) - H_{az} \tag{B-3}$$

$$F_{44} = F_{46} = 0 \tag{B-4}$$

$$F_{45} = 2\omega \tag{B-5}$$

$$F_{51} = \mu y/r^3\ (3x/r^2)(1+H_a) - H_{ax} \tag{B-6}$$

$$F_{52} = \mu/r^3\left[3(y/r)^2-1\right]\left[1+H_a\right] - yH_{ay} + \omega^2 \tag{B-7}$$

$$F_{53} = \mu y/r^3(3z/r^2)(1+H_a) - H_{az} \tag{B-8}$$

$$F_{54} = -2\omega \tag{B-9}$$

$$F_{55} = F_{56} = 0 \tag{B-10}$$

where the terms H_a, H_{ax}, H_{ay}, and H_{az} are given by

$$H_a = \frac{3J_2A_e^2}{2r^2}\left[1-5(z/r)^2\right] + \frac{5J_3A_e^3}{2r^3}\left[3-7(z/r)^2\right]z/r \qquad \text{(B-11)}$$

$$H_{ax} = \frac{3J_2A_e^2x}{r^4}\left[10(z/r)^2-1\right] + \frac{15J_3xzA_e^3}{r^6}\left[7(z/r)^2-2\right] \qquad \text{(B-12)}$$

$$H_{ay} = \frac{3J_2A_e^2y}{r^4}\left[10(z/r)^2-1\right] + \frac{15J_3yzA_e^3}{r^6}\left[7(z/r)^2-2\right] \qquad \text{(B-13)}$$

$$H_{az} = \frac{6J_2A_e^2z}{r^4}\left[5(z/r)^2-3\right] + \frac{15J_3A_e^3}{r^4}\left[0.5-5.5(z/r)^2+ 7(z/r)^4\right] \qquad \text{(B-14)}$$

$$F_{61} = \mu z/r^3 \quad (3x/r^2)(1+H_b) - H_{bx} \qquad \text{(B-15)}$$

$$F_{62} = \mu z/r^3 \; (3y/r^2)(1+H_b) - H_{by} \qquad \text{(B-16)}$$

$$F_{63} = \mu/r^3 \left[3(z/r)^2-1\right]\left[1+H_b\right] - zH_{bz} \qquad \text{(B-17)}$$

$$F_{64} = F_{65} = F_{66} = 0 \qquad \text{(B-18)}$$

where the terms H_b, H_{bx}, H_{by}, and H_{bz} are given by

$$H_b = \frac{3J_2A_e^2}{2r^2}\left[3-5(z/r)^2\right] + \frac{5J_3A_e^3}{2r^3}\left[6-7(z/r)^2\right]z/r \qquad \text{(B-19)}$$

$$H_{bx} = \frac{J_2A_e^2x}{r^4}\left[30(z/r)^2-9\right] + \frac{15J_3A_e^3xz}{r^6}\left[7(z/r)^2-4\right] \qquad \text{(B-20)}$$

$$H_{by} = \frac{J_2 A_e^2 y}{r^4}\left[30(z/r)^2-9\right] + \frac{15 J_3 A_e^3 yz}{r^6}\left[7(z/r)^2-4\right] \quad \text{(B-21)}$$

$$H_{bz} = \frac{6 J_2 A_e^2 z}{r^4}\left[5(z/r)^2-4\right] + \frac{15 J_3 A_e^3}{r^4}\left[1+7(z/r)^4-7.5(z/r)^2\right] \quad \text{(B-22)}$$

Appendix C

Measurement Matrix M

The measurement matrix M is made up of the partial derivatives of the measurements ρ, $\dot{\rho}$, A, and E with respect to the states, position and velocity.

$$M = \begin{bmatrix} M(1) \\ M(2) \\ M(3) \\ M(4) \end{bmatrix} = \begin{bmatrix} \frac{\partial \rho}{\partial x}, & \frac{\partial \rho}{\partial y} & \cdots & \frac{\partial \rho}{\partial \dot{z}} \\ \vdots & & & \vdots \\ \frac{\partial E}{\partial x} & \cdots & \cdots & \frac{\partial E}{\partial \dot{z}} \end{bmatrix} \tag{C-1}$$

where the terms M(1), M(2), M(3), and M(4) are given by

$$M(1) = \left[(x+X)/\rho,\ (y+Y)/\rho,\ (z+Z)/\rho,\ 0,\ 0,\ 0\right] \tag{C-2}$$

$$M(2) = \left[\frac{\dot{x}}{\rho} - \frac{\dot{\rho}(x+X)}{\rho^2},\ \frac{\dot{y}}{\rho} - \frac{\dot{\rho}(y+Y)}{\rho^2},\ \frac{\dot{z}}{\rho} - \frac{\dot{\rho}(z+Z)}{\rho^2},\ \frac{x+X}{\rho},\ \frac{y+Y}{\rho},\ \frac{z+Z}{\rho}\right] \tag{C-3}$$

$$M(3) = \left[1/(\rho^2 - z_T^2)\right]\left[-x_T\sin(\theta) - y_T\sin(\phi)\cos(\theta),\ x_T\cos(\theta) - y_T\sin(\phi)\sin(\theta),\ y_T\cos(\phi), 0, 0, 0\right] \tag{C-4}$$

$$M(4) = \left[1/(\rho^2 - z_T^2)^{1/2}\right]\left[\cos(\phi)\cos(\theta) - z_T(x-X)/\rho^2,\ \sin(\theta)\cos(\phi) - z_T(y-Y)/\rho^2,\ \sin(\phi) - z_T(z-Z)/\rho^2,\ 0,0,0\right] \tag{C-5}$$

Appendix D

Computer Program

The computer program listing is presented in this appendix, along with a common listing of the variables used in the program, and a sample of the required input data. The program is written in Fortran IV language for use on the IBM 7094 computer located at Wright-Patterson Air Force Base, Ohio.

*** COMMON LISTING ***

```
C(  1)          T            TIME
C(  2)
C(  3)          DT           INTEGRATION STEP SIZE
C(  4)          SQRMU        SQUARE ROOT OF MU
C(  5)          DT4          DT3*DT/4.0
C(  6)          WIEKE        ANGULAR ROTATATIONAL RATE OF EARTH
C(  7)          DT3          DT2*DT/3.0
C(  8)          DT2          DT*DT/2.0
C(  9)          VO           CIRCULAR SATELLITE SPEED
C( 10)          AE           RADIUS OF EARTH
C( 11)
C( 12)          MU           GRAVITATIONAL CONSTANT (=1.0)
C( 13)          WE,WIE       ANGULAR ROTATIONAL RATE OF EARTH
C( 14)          WE2,WIE2     WE*WE
C( 15)          F            FLATNESS COEFFICIENT
C( 16)          KE           CONSTANT
C( 17)          J2           CONSTANT
C( 18)          J3           CONSTANT
C( 19)          MUMET        CONSTANT (MU FOR METER UNITS)
C( 20)          TI           TIME INTERVAL MEASUREMENTS
C( 21)
C( 22)
C( 23)
C( 24)
C( 25)          EB           ELEVATION BIAS
C( 26)          AB           AZIMUTH BIAS
C( 27)          RB           RANGE BIAS
C( 28)          RDB          RANGE RATE BIAS
C( 29)          HOUR         HOUR OF OBSERVATION
C( 30)          MINUTE       MINUTE OF OBSERVATION
C( 31)          HP           HEIGHT AT PERIGEE
C( 32)          NO           NUMBER OF OBSERVATIONS
C( 33)
C( 34)
C( 35)
C( 36)
C( 37)
C( 38)
C( 39)
C( 40)
C( 41) THRU C( 49)           CLT(1,1) THRU CLT(3,3)
C( 50)
C( 51)          RHOM         RHO MEASURED
C( 52)          RHODM        RHO-DOT MEASURED
C( 53)          AM           AZIMUTH MEASURED
C( 54)          EM           ELEVATION MEASURED
C( 55)
C( 56)          RHOC         RHO CALCULATED
```

C(57)	RHODC	RHO-DOT CALCULATED
C(58)	AC	AZIMUTH CALCULATED
C(59)	EC	ELEVATION CALCULATED
C(60)		
C(61)	AD	AZIMUTH-DOT
C(62)	ED	ELEVATION-DOT
C(63)		
C(64)		
C(65)		
C(66)		
C(67)		
C(68)		
C(69)		
C(70)	PER	PERIOD
C(71)	A	SEMI-MAJOR AXIS
C(72)	ECC	ECCENTRICITY
C(73)	T	EPOCH TIME
C(74)	E	ECCENTRIC ANOMALY
C(75)	M	MEAN ANOMALY
C(76)	PDP,UDU	DOT PRODUCT OF UNIT VECTORS
C(77)	PDQ,UDV	DOT PRODUCT OF UNIT VECTORS
C(78)	ANGI	INCLINATION ANGLE
C(79)	ANGALN	ANGLE TO LINE OF NODES
C(80)	ANGAP	ANGLE OF ARGUMENT OF PERIGEE
C(81)	PX,UX	X-COMPONENT OF P OR U
C(82)	PY,UY	Y-COMPONENT OF P OR U
C(83)	PZ,UZ	Z-COMPONENT OF P OR U
C(84)	QX,VX	X-COMPONENT OF Q OR V
C(85)	QY,VY	Y-COMPONENT OF Q OR V
C(86)	QZ,VZ	Z-COMPONENT OF Q OR V
C(87)	WX	X-COMPONENT OF W
C(88)	WY	Y-COMPONENT OF W
C(89)	WZ	Z-COMPONENT OF W
C(90)		
C(91)	X	POSITION OF SATELLITE IN METERS
C(92)	Y	POSITION OF SATELLITE IN METERS
C(93)	Z	POSITION OF SATELLITE IN METERS
C(94)	VX	VELOCITY OF SATELLITE IN METERS/SEC
C(95)	VY	VELOCITY OF SATELLITE IN METERS/SEC
C(96)	VZ	VELOCITY OF SATELLITE IN METERS/SEC
C(97)	R	MAGNITUDE OF SATELLITE POS VECTOR
C(98)	V	MAGNITUDE OF SATELLITE VEL VECTOR
C(99)		
C(100)		
C(101)	X	POSITION OF SATELLITE IN EARTH RADII
C(102)	Y	POSITION OF SATELLITE IN EARTH RADII
C(103)	Z	POSITION OF SATELLITE IN EARTH RADII
C(104)	VX	VELOCITY OF SATELLITE IN EARTH UNITS
C(105)	VY	VELOCITY OF SATELLITE IN EARTH UNITS
C(106)	VZ	VELOCITY OF SATELLITE IN EARTH UNITS
C(107)		
C(108)		

C(109)		
C(110)		
C(111)	XD	DERIVATIVES FOR INTEGRATION
C(112)	YD	DERIVATIVES FOR INTEGRATION
C(113)	ZD	DERIVATIVES FOR INTEGRATION
C(114)	VXD	DERIVATIVES FOR INTEGRATION
C(115)	VYD	DERIVATIVES FOR INTEGRATION
C(116)	VZD	DERIVATIVES FOR INTEGRATION
C(117)		
C(118)		
C(119)		
C(120)		
C(121)		
C(122)		
C(123)	THEGRO	INITIAL GREENWICH TIME
C(124)	THETA	MEAN SIDEREAL TIME(GREENWICH)
C(125)		
C(126)	LAT	LATITUDE OF STATION
C(127)	LONG	LONGITUDE OF STATION
C(128)	HT	HEIGHT OF STATION
C(129)	TDOT	THETA-DOT
C(130)		
C(131)	XS	GEOCENTRIC COORD OF STATION IN E.R.
C(132)	YS	GEOCENTRIC COORD OF STATION IN E.R.
C(133)	ZS	GEOCENTRIC COORD OF STATION IN E.R.
C(134)	XVS	
C(135)	VYS	
C(136)	VZS	
C(137)	XS,XSM	GEOCENTRIC COORD OF STATION (M)
C(138)	YS,YSM	GEOCENTRIC COORD OF STATION (M)
C(139	ZS,ZSM	GEOCENTRIC COORD OF STATION (M)
C(140)		
C(141)	XVS	COMPONENT OF SATELLITE IN STN COORD
C(142)	YVS	COMPONENT OF SATELLITE IN STN COORD
C(143)	ZVS	COMPONENT OF SATELLITE IN STN COORD
C(144) THRU C(166)		NOT USED
C(167)	SECOND	SECOND OF OBSERVATION
C(168) THRU C(200)		NOT USED
C(201) THRU C(236)		F(1,1) THRU F(6,6)
C(237)		
C(238)	D	DETERMINANT OF PE
C(239)		
C(240)		
C(241) THRU C(264)		M(1,1) THRU M(4,6)
C(265)		
C(266)	R(1)	SIGMA**2 RHO
C(267)	R(2)	SIGMA**2 RHO-DOT
C(268)	R(3)	SIGMA**2 AZIMUTH
C(269)	R(4)	SIGMA**2 ELEVATION
C(270)		
C(271) THRU C(294)		K(1,1) THRU K(6,4)
C(295)		

C(296)	Z(1)	DELTA RHO
C(297)	Z(2)	DELTA RHO-DOT
C(298)	Z(3)	DELTA AZIMUTH
C(299)	Z(4)	DELTA ELEVATION
C(300) THRU C(340)		NOT USED
C(341) THRU C(376)		PF(1,1) THRU PF(6,6)
C(377)		
C(378)		
C(379)		
C(380)		
C(381)	DXEST(1)	ESTIMATE OF X ERROR
C(382)	DXEST(2)	ESTIMATE OF Y ERROR
C(383)	DXEST(3)	ESTIMATE OF Z ERROR
C(384)	DXEST(4)	ESTIMATE OF VX ERROR
C(385)	DXEST(5)	ESTIMATE OF VY ERROR
C(386)	DXEST(6)	ESTIMATE OF VZ ERROR
C(387) THRU C(400)		NOT USED
C(401) THRU C(436)		PP(1,1) THRU PP(6,6)
C(437) THRU C(472)		PHI(1,1) THRU PHI(6,6)
C(473) THRU C(500)		NOT USED
C(501)	STANR	STATION NUMBER
C(502)	SATNR	SATELLITE NUMBER
C(503)	YEAR	YEAR OF OBSERVATION
C(504)	DAY	DAY OF OBSERVATION

```
$IBFTC MAIN

      COMMON/AFIT/C(600)
      DIMENSION X(6),XER(6),D(600),XERR(75),YERR(75),
     1VXERR(75),VYERR(75),VZERR(75),TIME(75),TP1(75),
     2TP3(75),TP4(75),TP5(75),TP6(75),TP7(75),TP8(75),
     3TA(75),TE(75),TI(75),ZERR(50),TP2(75)
      EQUIVALENCE (C(001),T),(C(091),X),(C(101),XER),
     1(C(123),THETGO),(C(381),DXEST1),(C(382),DXEST2),
     2(C(124),THETA ),(C(384),DXEST4),(C(385),DXEST5),
     3(C(401),PP11),(C(408),PP22),(C(415),PP33),
     4(C(422),PP44),(C(429),PP55),(C(436),PP66),
     5(C(71),A),(C(72),B),(C(73),AI),(C(383),DXEST3),
     6(C(386),DXEST6),(C(403),PP31),(C(424),PP64)
     7(C(010),AE),(C(007),VO)
  601 FORMAT(1H1,5X,14HCOMMON LISTING,5X,7HTIME = ,F15.7/)
  600 FORMAT(I5,2X,1P3E15.7)
C
C     TCTR IS A TIME COUNTER
C
  100 TCTR = 0.0
      NT = 1
      CALL ZERO
      CALL INPUT
      CALL STAT
      CALL EOCOMP
      CALL ELEMTS
      WRITE (6,601) T
      WRITE (6,600) (I,C(I),C(I+1),C(I+2),C(I+3),C(I+4),
     1C(I+5),C(I+6),C(I+7),I = 1,472,8)
      KOUNT=0
      KTCK = 60
C
C     NO REPRESENTS THE NUMBER OF OBSERVATIONS TO BE
C     READ IN. NONO IS USED TO DETERMINE WHEN THERE ARE
C     NO MORE MEASUREMENTS. KTCK DEPENDS UPON THE
C     MEASUREMENT INTERVALS OF THE RADAR STATIONS
C
      NONO = (NO-1)*KTCK
      GO TO 2
C
C     IN=0 (A BLANK CARD) IMPLIES THAT AN OBSERVATION IS
C     AVAILABLE AND THIS DATA IS READ IN. OTHERWISE
C     INTEGRATION OF THE EQUATIONS OF MOTION CONTINUES
C     FOR ANOTHER TIME INTERVAL.
C
    1 READ (5,500) IN
  500 FORMAT (I1)
      IF(IN)4,11,4
   11 CALL INPUTA
    2 CALL KALMAN
```

```
      DO 16 I=1,3
      J=I+3
      XER(I)=X(I)/AE
   16 XER(J)=X(J)/VO
      CALL ELEMTS
      CALL OUTPTA
      CALL TRAJI
C
C     THE FOLLOWING STATEMENTS (DOWN TO TI(NT)=AI) ARE
C     USED FOR TEMPORARY STORAGE OF DATA FOR USE
C     IN THE PLOTTING ROUTINE.
C
4     TIME(NT) = TCTR
      TCTR = TCTR + 6.0
      XERR( NT ) = DXEST1
      YERR( NT ) = DXEST2
      ZERR( NT ) = DXEST3
      VXERR( NT ) = DXEST4
      VYERR( NT ) = DXEST5
      VZERR( NT ) = DXEST6
      TP1(NT) = PP11
      TP2(NT) = PP22
      TP3(NT) = PP33
      TP4(NT) = PP44
      TP5(NT) = PP55
      TP6(NT) = PP66
      TP7(NT) = PP31
      TP8(NT) = PP64
      TA(NT) = A
      TE(NT) = E
      TI(NT) = AI
      NT = NT + 1
      WRITE (6,601) T
      WRITE (6,600) (I,C(I),C(I+1),C(I+2),C(I+3),C(I+4),
     1C(I+5),C(I+6),C(I+7),I = 1,472,8)
    3 CALL UPCOV
      CALL TRAJ
      KOUNT=KOUNT+1
      IF (KOUNT.LT.KTCK) GO TO 3
      KTCK = KTCK + 60
C
C     THIS STATEMENT UPDATES GREENWICH SIDEREAL
C     TIME EVERY MEASUREMENT INTERVAL
C
      THETA = THEGRO + 0.004375269*TCTR/60.0
      IF (KOUNT.LE.NONO) GO TO 1
      DO 17 I=1,3
      J=I+3
      XER(I)=X(I)/AE
   17 XER(J)=X(J)/VO
      CALL ELEMTS
      CALL OUTPTA
```

```
      CALL PLOTS(D,600)
      CALL DRAW(XERR,YERR,ZERR,XERR,TIME,NO,3,3)
      CALL DRAW (VXERR,VYERR,VZERR,VXERR,TIME,NO,3,3)
      CALL DRAW(TP1,TP2,TP3,TP7,TIME,NO,4,4)
      CALL DRAW(TP4,TP5,TP6,TP8,TIME,NO,4,4)
      CALL DRAW(TA,TE,TI,TA,TIME,NO,3,1)
      CALL PLOTE
       GO TO 100
      END
```

****

```
$IBFTC ZERO.
      SUBROUTINE ZERO
C
C     THIS SUBROUTINE SETS THE 600 COMMON STORAGE
C     LOCATIONS TO ZERO AND DEFINES ALL NECESSARY CONSTANTS
C
      COMMON/AFIT/C(600)
      REAL MU,MUMET,J2,J3,KE
      EQUIVALENCE       (C(219),F14     ),(C(226),F25     ),
     1(C(233),F36     ),(C(017),J2      ),(C(018),J3      ),
     2(C(015),F       ),(C(010),AE      ),(C(009),VO      ),
     3(C(012),MU      ),(C(129),TDOT    ),(C(016),KE      ),
     4(C(013),WIE     ),(C(003),DT      ),(C(019),MUMET   ),
     5                 (C(014),WIE2    ),(C(008),DT2     ),
     6                 (C(007), DT3),(C(006),WIEKE)
      NAMELIST/NAMES/DT
      DO 1 I = 1,600
    1 C(I)=-0.
      READ (5,NAMES)
      F14=1.0
      F25=1.0
      F36=1.0
      MU=1.0
      J2=1082.645E-06
      J3=-2.546E-06
      F=1.0/298.25
      TDOT=0.05883447
      MUMET=3.986032E+14
      WIE = 7.29215E-5
C
C     MULTIPLY WIE BY CANONICAL TIME UNIT FOR USE IN
C     SUBROUTINE ELEMENTS.
C
      WIEKE=WIE*806.81364
      WIE2=WIE*WIE
      KE=4.2608430
      AE=6378160.0
      VO=7.905376E+03
      DT2=DT*DT/2.0
      DT3 = DT2*DT/3.0
```

```
      RETURN
      END
                   ******************************
$IBFTC INPUTZ
      SUBROUTINE INPUT
C
C     THIS SUBROUTINE READS IN THE FIRST SET OF DATA
C     WHICH IS USED IN SUBROUTINE FOCOMP TO ESTABLISH
C     INITIAL NOMINAL TRAJECTORY
C
      COMMON/AFIT/C(600)
      REAL LAT,LONG,MINUTE
      INTEGER SATNR,STANR,YEAR,DAY,HOUR
      EQUIVALENCE       (C(001),T        ),(C(003),DT       ),
     1(C(128),HT      ),(C(126),LAT      ),(C(124),THETA    ),
     2(C(054),EL      ),(C(053),AZ       ),(C(051),RHO      ),
     3(C(061),ADOT    ),(C(062),EDOT     ),(C(341),P11      ),
     4(C(355),P33     ),(C(362),P44      ),(C(369),P55      ),
     5(C(266),R1      ),(C(267),R2       ),(C(268),R3       ),
     6(C(501),STANR   ),(C(502),SATNR    ),(C(503),YEAR     ),
     7(C(029),HOUR    ),(C(030),MINUTE),(C(167),SECOND),
     8(C(025),EB),(C(026),AB),(C(027),RB),(C(028),RDB),
     9(C(127),LONG    ),(C(052),RHODOT),(C(348),P22     ),
     A(C(370),P66     ),(C(269),R4       ),(C(504),DAY      ),
     B(C(123),THEGRO),(C(020),TI        ),(C(032),NO       )
      NAMELIST/NAME1/EB,AB,RB,RDB/NAME2/P11,P22,P33,P44,
     1P55,P66/NAME3/R1,R2,R3,R4/NAME6/HR,PINUT,SEC,NO,TI
      READ (5,NAME1)
      READ (5,NAME2)
      READ (5,NAME3)
C
C     R1 THROUGH R4 (COMPUTED BELOW) ARE EQUAL TO THE
C     SIGMAS SQUARED.
C
      R1 = R1**2
      R2 = R2**2
      R3 = (R3/57.3)**2
      R4 = (R4/57.3)**2
      READ(5,NAME6)
      WRITE(6,NAME6)
      READ (5,501) LAT,LONG,HT
      READ (5,502) SATNR,STANR,YEAR,DAY,HOUR,MINUTE,SECOND,
     1EL,AZ,RHO,INDEX,RHODOT,EDOT,ADOT
     1 INDEX,RHODOT,EDOT,ADOT
C
C     THETA IS THE MEAN SIDEREAL TIME AT GREENWICH.
C
      RHR = HOUR
      THETA=(HR+RHR +(PINUT+MINUTE)/60.0+(SEC+SECOND)/
     13600.0)*15.0
```

```
C
C      CONVERT THETA FROM DEGREES TO RADIANS.
C
       THETA = THETA*1.7453293E-2
       THEGRO = THETA
       HOUR = RHR
       GO TO (1,2,3,4),INDEX
  1    RHO = RHO*10.0
       GO TO 100
  2    RHO = RHO*100.0
       GO TO 100
  3    RHO = RHO*1000.0
       GO TO 100
  4    RHO = RHO*10000.0
 100  WRITE (6,600) SATNR,STANR,YEAR,DAY,HOUR,MINUTE,
     2SECOND,
     1LAT,THETA,LONG,HT,EL,AZ,RHO,RHODOT,EDOT,ADOT,T,DT
 501   FORMAT (4X,F7.4,F8.4,F6.0)
 502   FORMAT (1X,I5,I3,I2,I3,I2,F2.0,F5.3,F6.4,1X,F7.4,1X,
     1F7.2,I1,1X,F7.2,1X,F5.4,1X,F5.4)
 600   FORMAT(1H1,32X,33HINPUT DATA FOR INITIAL TRAJECTORY/
     138X,19HSATELLITE NUMBER = ,I5,10X,
     B17HSTATION NUMBER = ,
     2I3/43X,4HYEAR,5X,3HDAY,5X,4HHOUR,5X,6HMINUTE,5X,
     36HSECOND/44X,I2,6X,I3,6X,I2,8X,I2,7X,F6.3//
     93X,11HLATITUDE = ,E15.7,5X,10HTHETA-G = ,E15.7,5X,
     412HLONGITUDE = ,E15.7,5X,17HSTATION HEIGHT = ,F6.0/3X
     512HELEVATION = ,E15.7,4X,10HAZIMUTH = ,E15.7,5X,
     66HRHO = ,E15.7,11X,10HRHO-DOT = ,E15.7/3X,
     A17HELEVATION RATE = ,E15.7,29X,
     715HAZIMUTH RATE = ,E15.7//3X,16HSTARTING TIME = ,
     8F4.2/3X,24HINTEGRATION STEP SIZE = ,F4.2)
C
C      STATION BIAS CORRECTION.
C
       EL = EL + EB
       AZ = AZ + AB
       RHO = RHO + RB
       RHODOT = RHODOT + RDB
       RETURN
       END
```

****

```
$IBFTC INPUTB
       SUBROUTINE INPUTA
C
C      THIS SUBROUTINE READS IN ALL MEASUREMENTS
C      SUBSEQUENT TO THE FIRST MEASUREMENT
C
       COMMON/AFIT/C(600)
       EQUIVALENCE    (C( 51),RHO    ),(C( 52),RHODOT),
```

```
     1(C(053),AZ      ),(C( 54),EL       ),(C( 61),AZDOT ),
     2(C(025),EB),(C(026),AB),(C(027),RB),(C(028),RDB),
     3(C(062),ELDOT )
      READ(5,500)EL,AZ,RHO,INDEX,RHODOT,ELDOT
 500  FORMAT(23X,F6.4,1X,F7.4,1X,F7.2,I1,1X,F7.2,1X,F5.4)
      GO TO (1,2,3,4),INDEX
 1    RHO = RHO*10.0
      GO TO 100
 2    RHO = RHO*100.0
      GO TO 100
 3    RHO = RHO*1000.0
      GO TO 100
 4    RHO = RHO*10000.0
C
C     STATION BIAS CORRECTION.
C
 100  EL = EL + EB
      AZ = AZ + AB
      RHO = RHO + RB
      RHODOT = RHODOT + RDB
      RETURN
      END

                 ******************************

$IBFTC OUTPTZ
      SUBROUTINE OUTPTA
C
C     THIS SUBROUTINE WRITES THE CURRENT VALUES OF
C     THE POSITION AND VELOCITY OF THE SATELLITE
C     ALONG WITH ITS ORBITAL ELEMENTS
C
      COMMON/AFIT/C(600)
      EQUIVALENCE (C(020),TI)
      C(1)=C(1)-TI
      WRITE (6,600) C(1),C(128),C(124),C(126),C(127),C(54),
     1C(51),C(62),C(53),C(52),C(61)
      C(1)=C(1)+TI
      WRITE (6,601) C(1),C(91),C(101),C(94),C(104),C(92),
     3C(102)C(95),
     1C(105),C(93),C(103),C(96),C(106),C(97),C(98),C(51),
     2C(52),C(53),C(54),C(56),C(57),C(58),C(59)
      WRITE (6,602) C(1),C(71),C(72),C(78),C(79),C(80),
     1C(74),C(75),C(73)
 600  FORMAT(1H1,4X,10HINPUT DATA,5X,7HTIME = ,F15.7//56X,
     118HSTATION PARAMETERS/8X,9HHEIGHT = ,E15.7,5X,
     78HTHETA = ,E15.7,
     25X,11HLATITUDE = ,E15.7,5X,12HLONGITUDE = ,E15.7//56X
     318HRADAR OBSERVATIONS/15X,12HELEVATION = ,E15.7,5X,
     414HSLANT RANGE = ,E15.7,10X,17HELEVATION RATE = ,
     8E15.7/15X,
     510HAZIMUTH = ,E15.7,7X,19HSLANT RANGE RATE = ,E15.7,
```

```
     65X,15HAZIMUTH RATE  =  ,E15.7///)
 601  FORMAT (4X,11HOUTPUT DATA,5X,7HTIME = ,F15.7//49X,
     131HSATELLITE POSITION AND VELOCITY//8X,
     27HX(M) = ,E15.7,5X,8HX(ER) = ,E15.7,5X,
     310HVX(M/S) = ,E15.7,5X,9HVX(EU) = ,E15.7/8X,
     47HY(M) = ,E15.7,5X,8HY(ER) = ,E15.7,5X,
     510HVY(M/S) = ,E15.7,5X,9HVY(EU) = ,E15.7/8X,
     67HZ(M) = ,E15.7,5X,8HZ(ER) = ,E15.7,5X,
     710HVZ(M/S) = ,E15.7,5X,9HVZ(EU) = ,E15.7/8X,7HR(M) =
     8E15.7,33X,
     89HV(M/S) = ,E15.7/ 8X,8HRHO-M = ,E15.7,4X,
     C12HRHO-DOT-M = ,E15.7,
     91X,7HAZ-M = ,E15.7,8X,7HEL-M = ,E15.7/ 8X,8HRHO-C = ,
     DE15.7,4X,
     A12HRHO-DOT-C = ,E15.7,1X,7HAZ-C = ,E15.7,8X,7HEL-C =
     FE15.7///)
 602  FORMAT(4X,16HORBITAL ELEMENTS,5X,7HTIME = ,F15.7//5X,
     34HA = ,E15.7,
     15X,6HECC = ,E15.7,5X,4HI = ,E15.7,5X,5HLN = ,E15.7,
     45HAP = ,
     2E15.7/5X,4HE = ,E15.7,5X,4HM = ,E15.7,7X,5HET = ,
     5E15.7)
      RETURN
      END
```

****

```
$IBFTC COV.
      SUBROUTINE UPCOV
C
C     THIS SUBROUTINE UPDATES THE ERROR COVARIANCE MATRIX
C     BY USE OF THE STATE TRANSITION MATRIX
C
      COMMON/AFIT/C(600)
      DIMENSION PP(6,6),PE(6,6),F(6,6),PAD66(6,6),PHI(6,6),
     1PAD661(6,6),PHIT(6,6)
      EQUIVALENCE      (C(201),F      ),(C(238),D      ),
     1(C(341),PE     ),(C(401),PP     ),(C(003),DT     ),
     2(C(008),DT2    ),(C(007),DT3    ),(C(437),PHI)
C
C     COMPUTE THE SYSTEM DESCRIPTION MATRIX - F
C
      CALL SDM
C
C     COMPUTE STATE TRANSITION MATRIX - PHI  AND  PHIT
C
      CALL MPRD (F,F,PAD66,6,6,0,0,6)
      CALL MPRD(F,PAD66,PAD661,6,6,0,0,6)
      DO 11 I=1,6
      DO 10 J=1,6
   10 PHI(I,J)=F(I,J)*DT+PAD66(I,J)*DT2+PAD661(I,J)*DT3
   11 PHI(I,I)=PHI(I,I)+1.0
```

```
      CALL MTRA (PHI,PHIT,6,6,0)
C
C     UPDATE FILTER ESTIMATION COVARIANCE MATRIX - PF
C
      CALL MPRD (PHI,PP,PAD66,6,6,0,0,6)
      CALL MPRD (PAD66,PHIT,PF,6,6,0,0,6)
      D=DET(PF,6)
      DO 23 I=1,6
      DO 23 J=1,6
   23 PP(I,J)=PF(I,J)
      RETURN
      END

               ******************************

$IBFTC EQCOM.
      SUBROUTINE EQCOMP
C
C     THIS SUBROUTINE WHEN GIVEN RADAR MEASUREMENTS
C     COMPUTES VALUES FOR POSITION AND VELOCITY OF THE
C     SATELLITE IN THE ROTATING GEOCENTRIC FRAME
C
      COMMON/AFIT/C(600)
      REAL LXR,LYR,LZR,LXDR,LYDR,LZDR,L1,L2,L3,KE,LONG,LONGR
      EQUIVALENCE     (C(053),A      ),(C(054),E       ),
     1(C(051),RHO    ),(C(061),ADOT   ),(C(062),EDOT    ),
     2(C(052),RHODOT),(C(126),PHI    ),(C(127),LONG),
     3(C(128),HEIGHT),(C(131),CX     ),(C(132),CY      ),
     4(C(133),CZ     ),(C(101),RX     ),(C(102),RY      ),
     6(C(103),RZ     ),(C(104),RXD    ),(C(105),RYD     ),
     7(C(106),RZD    ),(C(010),AE     ),(C(016),KE      ),
     8(C(129),TDOT   ),(C(091),XMER   ),(C(092),YMER    ),
     9(C(093),ZMER   ),(C(094),XDMER  ),(C(095),YDMER   ),
     A(C(096),ZDMER  ),(C(137),XSMR   ),(C(138),YSMR    ),
     B(C(139),ZSM    ),(C(015),F),(C(9),VO),(C(13),WE),
      DATA CDTR/1.7453293E-2/
      PHIR=PHI*CDTR
      AR=A*CDTR
      ER=E*CDTR
      LONGR = LONG*CDTR
      SG = SIN(LONGR)
      CG = COS(LONGR)
      SP=SIN(PHIR)
      CP=COS(PHIR)
      SE=SIN(ER)
      CE=COS(ER)
      SA=SIN(AR)
      CA=COS(AR)
      RHOF = RHO/AE
      RHOD = RHODOT/VO
      ED=EDOT*60.0/KE
      AD=ADOT*60.0/KE
```

```
      ADCE  =  AD
      H=HEIGHT/AE
      SEE=1.0/SQRT(1.0-(2.0*F-F*F)*SP*SP)
      S=SEE*(1.0-F)**2
C
C     COMPUTE COORDINATES OF STATION IN ROTATING FRAME
C
       CX   =   -(SEE+H)*CG*CP
        CY  =   -(SEE+H)*CP*SG
       CZ  =   -(S+H)*SP
       XSMR   =  CX*AE
       YSMR  =  CY*AE
       ZSM  =  CZ*AE
C
C     COMPUTE VECTOR L IN ROTATING FRAME
C
 200  L1=SP*CE*CA
      L2=CP*SE
      L3=CE*SA
      LXR  =  CG*(-L1+L2)  -  SG*L3
      LYR  =  SG*(-L1+L2)  +  CG*L3
      LZR  =  CP*CE*CA+SP*SE
C
C     COMPUTE VECTOR L-DOT IN ROTATING FRAME
C
      LXDR  =  CG*SP*SA*ADCE  +  CG*SP*SE*CA*ED    + CG*CP*CE*ED
     1-SG*CA*ADCE  +  SG*SA*SE*ED
      LYDR  =  SG*SP*SA*ADCE  +  SG*SP*SE*CA*ED    + SG*CP*CE*ED
     1+CG*CA*ADCE  -  CG*SE*SA*ED
      LZDR  =-CP*SA*ADCE-CP*CA*SE*ED  +SP*CE*ED
C
C     COMPUTE COORDINATES OF OBJECT WITH RESPECT TO ROTATING
C     SYSTEM
C
      RX  =  RHOE*LXR  -  CX
      RY  =  RHOE*LYR  -  CY
      RZ  =  RHOE*LZR  -  CZ
C
C     COMPUTE VELOCITY OF OBJECT WITH RESPECT TO ROTATING
C     SYSTEM
C
      RXD  =  RHOE*LXDR  +  RHOD*LXR
      RYD  =  RHOE*LYDR  +  RHOD*LYR
      RZD  =  RHOE*LZDR  +  RHOD*LZR
C     CONVERT FROM EARTH UNITS TO METERS
      XMER  =  RX*AE
      ZMER  =  RY*AE
      ZMER  =  RZ*AE
C
C     CONVERT FROM EARTH UNITS TO METERS/SEC
C
      XDMER  =  RXD*VO
```

```
      YDMER = RYD*VO
      ZDMER = RZD*VO
      RETURN
      END

                     ******************************

$IBFTC ELEMT.1
      SUBROUTINE ELEMT1
C
C     THIS SUBROUTINE WHEN GIVEN THE EQUATORIAL COMPONENTS
C             X, Y, Z, XD, YD, ZD
C     COMPUTES THE ORBITAL ELEMENTS  -
C             A, ECC, T, I, ALN, AN, E, M
C     UNIT VECTORS-P,Q,W- ARE USED
C
      COMMON /AFIT/ C(600)
      REAL I,MU,M,KE
      EQUIVALENCE      (C(101),X       ),(C(102),Y       ),
     1(C(103),Z      ),(C(104),XD      ),(C(105),YD      ),
     2(C(106),ZD     ),(C(071),A       ),(C(072),ECC     ),
     3(C(073),T      ),(C(074),E       ),(C(075),M       ),
     4(C(016),KE     ),(C(012),MU      ),(C(076),PDP     ),
     5(C(077),PDO    ),(C(078),ANGI    ),(C(079),ANGALN),
     6(C(080),ANGAP  ),(C(081),PX      ),(C(082),PY      ),
     7(C(083),PZ     ),(C(084),QX      ),(C(085),QY      ),
     8(C(086),QZ     ),(C(087),WX      ),(C(088),WY      ),
     9(C(089),WZ     ),(C(070),PER     ),(C(124),THEGR   ),
     A(C(006),WIEKE  ),(C(004),SQRMU),(C(010),AE        ),
     B(C(031),HP     )
      DATA CRTD/57.295780/
C
C     TRANSFER X,Y,Z,XD,YD,ZD, FROM ROTATING FRAME TO
C     INERTIAL FRAME
C
      ST = SIN(THEGR)
      CT = COS(THEGR)
      XI = X*CT - Y*ST
      YI = X*ST + Y*CT
      XDI = XD*CT - YD*ST - WIEKE*YI
      YDI = XD*ST + YD*CT + WIEKE*XI
      X = XI
      Y = YI
      XD = XDI
      YD = YDI
C
C     COMPUTE THE SEMI-MAJOR AXIS  -  A
C
 10   RO  = SQRT(X*X + Y*Y + Z*Z)
      VSQ=XD*XD+YD*YD+ZD*ZD
      SQRMU=SQRT(MU)
      AREC=(2./RO)-(VSQ/MU)
```

```
      A=1.0/AREC
C
C     COMPUTE THE PERIOD
C
      PER=6.28318*A*SQRT(A**3/MU)/KE
      Q = A*(1.0-ECC)
      HP = AE*(Q-1.0)
C
C     COMPUTE THE ECCENTRICITY  -  ECC
C
      DD=SQRMU*((1./RO)-AREC)
      RD=SQRT(V*Q)
      D=(X*XD+Y*YD+Z*ZD)/SQRMU
      EC=RO*DD/SQRMU
      ES=D/SQRT(A)
      ECC=SQRT(ES**2+EC**2)
C
C     COMPUTE THE ECCENTRIC ANOMALY  -  E
C
      E=ATAN2(ES,EC)
C
C     COMPUTE THE MEAN ANOMALY  -  M
C
      M=E-ES
C
C     COMPUTE THE EPOCH TIME  -  T
C
      T=M*SQRT(A**3)/(SQRMU*KE)
C
C     COMPUTE THE COMPONENTS OF P
C
      AX=(DD*X-D*XD)/SQRMU
      AY=(DD*Y-D*YD)/SQRMU
      AZ=(DD*Z-D*ZD)/SQRMU
      PX=AX/ECC
      PY=AY/ECC
      PZ=AZ/ECC
C
C     COMPUTE THE COMPONENTS OF Q
C
      SP=A*(1.-ECC*ECC)
      H=RO-SP
      HD=SQRMU*D/RO
      BX=(HD*X-H*XD)/SQRMU
      BY=(HD*Y-H*YD)/SQRMU
      BZ=(HD*Z-H*ZD)/SQRMU
      QD=1.0/(ECC*SQRT(SP))
      QX=BX*QD
      QY=BY*QD
      QZ=BZ*QD
C
C     COMPUTE THE COMPONENTS OF W
```

```
C
      HX=(Y*ZD-Z*YD)/SQRMU
      HY=(Z*XD-X*ZD)/SQRMU
      HZ=(X*YD-Y*XD)/SQRMU
      SRSP=SQRT(SP)
      WX=HX/SRSP
      WY=HY/SRSP
      WZ=HZ/SRSP
C
C     COMPUTE THE ANGLES  -  I, ALN, AP
C
      I=ARCOS(WZ)
      ALN=ATAN2(WX,-WY)
      AP=ATAN2(PZ,QZ)
      ANGI=I*CRTD
      IF(ANGI.LT.0.0)ANGI=ANGI+360.0
      ANGALN=ALN*CRTD
      IF(ANGALN.LT.0.0)ANGALN=ANGALN+360.0
      ANGAP=AP*CRTD
      IF(ANGAP.LT.0.0)ANGAP=ANGAP+360.0
      QDQ=QX*QX+QY*QY+QZ*QZ
      QDP=QX*PX+QY*PY+QZ*PZ
      RETURN
      END

                    *******************************

$IBFTC ELFMT.2
      SUBROUTINE ELFMT2
C
C     THIS SUBROUTINE WHEN GIVEN THE EQUATORIAL COMPONENTS
C              X, Y, Z, XD, YD, ZD
C     COMPUTES THE ORBITAL ELEMENTS  -
C              A, ECC, T, I, ALN, AN, E, M
C     UNIT VECTORS-U,V,W- ARE USED
C
      COMMON/AFIT/C(600)
      REAL I,MU,M,KE
      EQUIVALENCE      (C(101),X         ),(C(102),Y       ),
     1(C(103),Z        ),(C(104),XD       ),(C(105),YD      ),
     2(C(106),ZD       ),(C(071),A        ),(C(072),ECC     ),
     3(C(073),T        ),(C(074),E        ),(C(075),M       ),
     4(C(016),KE       ),(C(012),MU       ),(C(076),UDU     ),
     5(C(077),UDV      ),(C(078),ANGI     ),(C(079),ANGALN  ),
     6(C(080),ANGAP    ),(C(081),UX       ),(C(082),UY      ),
     7(C(083),UZ       ),(C(084),VX       ),(C(085),VY      ),
     8(C(086),VZ       ),(C(087),WX       ),(C(088),WY      ),
     9(C(089),WZ       ),(C(070),PER      ),(C(124),THEGR   ),
     A(C(006),WIEKE    ),(C(004),SQRMU),(C(010),AE        ),
     B(C(031),HP       )
      DATA CRTD/57.295780/
C
```

```
C       TRANSFER X,Y,Z,XD,YD,ZD, FROM ROTATING FRAME TO
C       INERTIAL FRAME
C
        CT = COS(THEGR)
        ST = SIN(THEGR)
        XI = X*CT - Y*ST
        YI = X*ST + Y*CT
        XDI = XD*CT - YD*ST - WIEKE*YI
        YDI = XD*ST + YD*CT + WIEKE*XI
        X = XI
        Y = YI
        XD = XDI
        YD = YDI
   10   RO  = SQRT(X*X + Y*Y + Z*Z)
        SQRMU=SQRT(MU)
        HX = Y*ZD-Z*YD
        HY = Z*XD-X*ZD
        HZ = X*YD-Y*XD
        P = HX*HX+HY*HY+HZ*HZ
        PSQR = SQRT(P)
        WX = HX/PSQR
        WY = HY/PSQR
        WZ = HZ/PSQR
        UX = X/RO
        UY = Y/RO
        UZ = Z/RO
        VX = WY*UZ-WZ*UY
        VY = WZ*UX-WX*UZ
        VZ = WX*UY-WY*UX
        RD = (X*XD+Y*YD+Z*ZD)/RO
        ESV = RD*PSQR
        ECV = (P/RO)-1.0
C
C       COMPUTE THE ECCENTRICITY  -  ECC
C
        ECC = SQRT(ESV**2+ECV**2)
C
C       COMPUTE THE SEMI-MAJOR AXIS  -  A
C
        A = P/(1.0-ECC**2)
        Q = A*(1.0-ECC)
        HP = AE*(Q-1.0)
C
C       COMPUTE THE PERIOD
C
   12   PER=6.2831854*SQRT(A**3/MU)/KE
        AX = UX*ECV-VX*ESV
        AY = UY*ECV-VY*ESV
        AZ = UZ*ECV-VZ*ESV
        SWZ = SQRT(1.0-WZ**2)
        AXN = (-AX*WY+AY*WX)/SWZ
        AYN = ((-WZ*(AX*WX+AY*WY))/SWZ)+AZ*SWZ
```

```
C
C     COMPUTE THE ECCENTRIC ANOMALY  -  E
C
      ESE =((RO*SQRT(1.0-ECC**2))*ESV)/P
      ECE = (ECV+ECC**2)/(1.0+ECV)
      E = ATAN2(ESE,ECE)
C
C     COMPUTE THE MEAN ANOMALY  -  M
C
      M = E-ESE
C
C     COMPUTE THE EPOCH TIME  -  T
C
      T=M*SQRT(A**3)/(SQRMU*KE)
C
C     COMPUTE THE ANGLES  -  I, ALN, AP
C
      I=ARCOS(WZ)
      ALN=ATAN2(WX,-WY)
      AP = ATAN2(AYN,AXN)
      ANGI=I*CRTD
      IF(ANGI.LT.0.0)ANGI=ANGI+360.0
      ANGALN=ALN*CRTD
      IF(ANGALN.LT.0.0)ANGALN=ANGALN+360.0
      ANGAP=AP*CRTD
      IF(ANGAP.LT.0.0)ANGAP=ANGAP+360.0
      UDU = UX*UX+UY*UY+UZ*UZ
      UDV = UX*VX+UY*VY+UZ*VZ
      RETURN
      END

                *****************************

$IBFTC STAT.
      SUBROUTINE STAT
C
C     THIS SUBROUTINE WHEN GIVEN THE LATITUDE AND
C     LONGITUDE OF A STATION COMPUTES THE ELEMENTS
C     OF COORDINATE TRANSFORMATION MATRIX
C
      COMMON/AFIT/C(600)
      REAL LAT,LONG,LATR,LONGR
      EQUIVALENCE     (C(126),LAT   ),(C(127),LONG   ),
     1(C(047),CLT13 ),(C(041),CLT11 ),(C(044),CLT12 ),
     2(C(048),CLT23 ),(C(042),CLT21 ),(C(045),CLT22 ),
     3(C(049),CLT33 ),(C(043),CLT31 ),(C(046),CLT32 ),
      DATA CDTR/1.745329E-2/
      LATR=LAT*CDTR
      LONGR=LONG*CDTR
      SLONG=SIN(LONGR)
      CLONG=COS(LONGR)
      SLAT=SIN(LATR)
```

```
      CLAT=COS(LATR)
      CLT11=-SLAT*CLONG
      CLT21=-SLAT*SLONG
      CLT31=CLAT
      CLT12=SLONG
      CLT22=-CLONG
      CLT32=0.0
      CLT13=CLAT*CLONG
      CLT23=CLAT*SLONG
      CLT33=SLAT
      RETURN
      END
```

****

```
$IBFTC MEAS.
      SUBROUTINE MEAS
C
C     THIS SUBROUTINE WHEN GIVEN THE POSITION AND
C     AND VELOCITY COMPUTES RANGE,RANGE RATE,AZIMUTH,
C     AND ELEVATION. IT ALSO COMPUTES THE ELEMENTS
C     OF THE MEASUREMENT MATRIX, M. THE VARIABLES COMPUTED
C     IN MEAS ARE USED IN SUBROUTINE KALMAN
C
      COMMON/AFIT/C(600)
      REAL LAT,LONG,LATR,LONGR,M(4,6)
      EQUIVALENCE     (C(091),X      ),(C(092),Y      ),
     1(C(093)(Z      ),(C(094),XD     ),(C(095),YD     ),
     2(C(096),ZD     ),(C(137),XS     ),(C(138),YS     ),
     3(C(139),ZS     ),(C(056),RHO    ),(C(057),RHODOT),
     4(C(013),WE     ),(C(141),XVS    ),(C(142),YVS    ),
     5(C(143),ZVS    ),(C(241),M      ),(C(126),LAT    ),
     6(C(127),LONG   ),(C(041),CLT11  ),(C(044),CLT12  ),
     7(C(047),CLT13  ),(C(042),CLT21  ),(C(045),CLT22  ),
     8(C(048),CLT23  ),(C(043),CLT31  ),(C(046),CLT32  ),
     9(C(049),CLT33  ),(C(058),AC     ),(C(059),EC     )
      DATA CDTR,CRTD/1.745329E-2,57.295779/
      LATR=LAT*CDTR
      LONGR=LONG*CDTR
      SLONG=SIN(LONGR)
      CLONG=COS(LONGR)
      SLAT=SIN(LATR)
      CLAT=COS(LATR)
      XR=X+XS
      YR=Y+YS
      ZR=Z+ZS
      XVS=CLT11*XR+CLT21*YR+CLT31*ZR
      YVS=CLT12*XR+CLT22*YR+CLT32*ZR
      ZVS=CLT13*XR+CLT23*YR+CLT33*ZR
C
C     CALCULATE RANGE, RANGE RATE, AZIMUTH, AND ELEVATION.
C
```

```
      RHO2=XR*XR+YR*YR+ZR*ZR
      RHO=SQRT(RHO2)
      RHODOT = (XR*XD + YR*YD + ZR*ZD)/RHO
      AR=ATAN2(-YVS,XVS)
      ER=ATAN2(ZVS,SQRT(RHO2-ZVS*ZVS))
      AC=AR*CRTD
      EC=ER*CRTD
C
C     COMPUTE THE MEASUREMENT MATRIX M.
C
      M(1,1)=XR/RHO
      M(1,2)=YR/RHO
      M(1,3)=ZR/RHO
      M(2,1) = (XD)/RHO - RHODOT*XR/RHO2
      M(2,2) = (YD)/RHO - RHODOT*YR/RHO2
      M(2,3)=ZD/RHO-RHODOT*ZR/RHO2
      M(2,4)=M(1,1)
      M(2,5)=M(1,2)
      M(2,6)=M(1,3)
      T1=1.0/(RHO2-ZVS*ZVS)
      M(3,1)=T1*(-XVS*SLONG-YVS*SLAT*CLONG)
      M(3,2)=T1*(+XVS*CLONG-YVS*SLAT*SLONG)
      M(3,3)=T1*(YVS*CLAT)
      T2=SQRT(T1)
      M(4,1)=T2*(CLAT*CLONG-ZVS*XR/RHO2)
      M(4,2)=T2*(CLAT*SLONG-ZVS*YR/RHO2)
      M(4,3)=T2*(SLAT-ZVS*ZR/RHO2)
      RETURN
      END

                  ******************************

$IBFTC TRAJ.
      SUBROUTINE TRAJI
C
C     THIS SUBROUTINE WHEN GIVEN INITIAL VALUES FOR THE
C     POSITION AND VELOCITY INTEGRATES THE NON-LINEAR
C     STATE EQUATION. THE INTEGRATION OCCURS BETWEEN
C     MEASUREMENTS
C
      COMMON/AFIT/C(600)
      DOUBLE PRECISION W
      DIMENSION D(6,5),W(6,5),Y(6),YD(6),XX(6)
      EQUIVALENCE      (C(003),H       ),(C(001),X       ),
     1(C(091),Y       ),(C(111),YD      )
      DATA M/6/
      K=0
      K2=0
      DO 10 I=1,M
 10   W(I,1)=DBLE(Y(I))
      CALL DERT
      DO 1 I=1,6
```

```
    1 D(I,5)=YD(I)
      RETURN
      ENTRY TRAJ
40    XC=X
      IF (K.NE.0) IF (K-2) 50,50,110
      XP=XC
      DO 45 I=1,M
45    W(I,5)=W(I,1)
50    K1=4-K
      DO 70 I=1,M
      DO 60 J=K1,4
60    D(I,J)=D(I,J+1)
      W(I,2)=H*D(I,4)
      W(I,1)=W(I,1)+.5D0*W(I,2)
70    Y(I)=SNGL(W(I,1))
      X=XC+.5*H
      CALL DERT
      DO 2 I=1,6
    2 D(I,5)=YD(I)
      DO 80 I=1,M
      W(I,3)=H*D(I,5)
      W(I,1)=W(I,1)+.5D0*(W(I,3)-W(I,2))
80    Y(I)=SNGL(W(I,1))
      CALL DERT
      DO 3 I=1,6
    3 D(I,5)=YD(I)
      DO 90 I=1,M
         98 -88-4(9,5)
      W(I,1)=W(I,1)+W(I,4)-.5D0*W(I,3)
90    Y(I)=SNGL(W(I,1))
      X=XC+H
      CALL DERT
      DO 4 I=1,6
    4 D(I,5)=YD(I)
      DO 100 I=1,M
      W(I,1)=W(I,1)-W(I,4)+.16666666666666667*(W(I,2)+2.D0*
     1(W(I,3)+W(I,4))+H*D(I,5))
100   Y(I)=SNGL(W(I,1))
      K=K+1
      K1=K
      CALL DERT
      DO 5 I=1,6
    5 D(I,5)=YD(I)
      RETURN
110   DO 130 I=1,M
      W(I,2)=W(I,1)
      DO 120 J=1,4
120   D(I,J)=D(I,J+1)
      W(I,3)=W(I,2)+.41666666666666667D-1*H*(55.*D(I,4)-59.*
     1D(I,0)+37.*D(I,2)-9.*D(I,1))
     1(I,2)-9.*D(I,1))
130   Y(I)=SNGL(W(I,2))
```

```
      X=XC+H
      CALL DERT
      DO 6 I=1,6
    6 D(I,5)=YD(I)
      DO 140 I=1,M
      W(I,1)=W(I,2)+.41666666666666667D-1*H*(9.*D(I,5)+19.*
     1D(I,4)-5.*D(I,3)+D(I,2))
     1,3)+D(I,2))
  140 Y(I)=SNGL(W(I,1))
      CALL DERT
      DO 7 I=1,6
    7 D(I,5)=YD(I)
      RETURN
      END
```

****

```
$IBFTC DERT.
C
C     SUBROUTINE DERT PROVIDES THE DERIVATIVE LIST FOR THE
C     INTEGRATION ROUTINE FOR THE TRAJECTORY GENERATION.
      SUBROUTINE DERT
      COMMON/AFIT/C(600)
      REAL MU,J2,J3
      EQUIVALENCE      (C(091),X      ),(C(092),Y      ),
     1(C(093),Z      ),(C(094),VX     ),(C(095),VY     ),
     2(C(096),VZ     ),(C(019),MU     ),(C(013),WIE    ),
     3(C(014),WIE2   ),(C(111),XD     ),(C(112),YD     ),
     4(C(113),ZD     ),(C(114),VXD    ),(C(115),VYD    ),
     5(C(116),VZD    ),(C(010),AE     ),(C(017),J2     ),
     6(C(018),J3     ),(C(097),R      ),(C(098),V      )
      R=SQRT(X*X+Y*Y+Z*Z)
      V=SQRT(VX*VX+VY*VY+VZ*VZ)
C
C     COMPUTE THE HARMONIC TERMS.
C
      A=MU/(R**3)
      AX=A*X
      AY=A*Y
      AZ=A*Z
      B=Z/R
      BB=B*B
      H=(J2*(AE**2))/(R**2)
      E=(J3*(AE**3))/(R**3)
      HA=1.5*H*(1.0-5.0*BB)+2.5*E*(3.0-7.0*BB)*B
      HB=1.5*H*(3.0-5.0*BB)+2.5*E*(6.0-7.0*BB)*B
      HA=HA+1.0
      HB=HB+1.0
      XD=VX
      YD=VY
      ZD=VZ
C
```

```
C      COMPUTE THE EQUATIONS OF MOTION.
C
       VXD=-AX*HA+2.0*WIE*VY+X*WIE2
       VYD=-AY*HA-2.0*WIE*VX+Y*WIE2
       VZD=-AZ*HP
       RETURN
       END

                  ******************************

$IBFTC KALM.
       SUBROUTINE KALMAN
C
C      THIS SUBROUTINE WHEN GIVEN A MEASUREMENT UPDATES
C      THE ERROR COVARIANCE MATRIX AND COMPUTES THE LATEST
C      OPTIMUM ESTIMATES OF STATE ERRORS
C
       COMMON/AFIT/C(600)
       REAL K(6,4),KT(4,6),M(4,6),MT(6,4)
       DIMENSION F(6,6),PHI(6,6),PP(6,6),PE(6,6),R(4),Z(4),
      3X(6),DXEST(6),
      1PAD66(6,6),PHIT(6,6),PAD64(6,4),PAD44(4,4),A(10),
      2PAD441(4,4),PAD66T(6,6),PAD661(6,6)
       EQUIVALENCE       (C(271),K      ),(C(241),M      ),
      1(C(003),DT     ),(C(201),F      ),(C(301),PHI    ),
      2(C(401),PP     ),(C(341),PE     ),(C(238),D      ),
      3(C(266),R      ),(C(296),Z      ),(C(091),X      ),
      4(C(281),DXEST  ),(C(056),RHOC   ),(C(057),RHODC  ),
      5(C(058),AC     ),(C(051),RHOM   ),(C(052),RHODM  ),
      6(C(053),AM     ),(C(059),EC     ),(C(054),EM     )
      7(C(061),AZDOT)
       NAMELIST/NAME4/M,MT,PE,PP,Z,A,K,IER
       DATA CDTR/1.7453293E-2/
C
C      UPDATE MEASUREMENT MATRIX - M
C
       CALL MEAS
C
C      COMPUTE FILTER GAIN MATRIX - K
C
       CALL MTRA (M,MT,4,6,0)
       CALL MPRD (PE,MT,PAD64,6,6,0,0,4)
       CALL MPRD (M,PAD64,PAD44,4,6,0,0,4)
       DO 12 I=1,4
    12 PAD44(I,I)=PAD44(I,I)+R(I)
       KK=1
       DO 200 I=1,4
       DO 200 J=1,I
       A(KK)=PAD44(J,I)
   200 KK=KK+1
       CALL SINV (A,4,1.0E-05,IER)
       KK=1
```

```
      DO 201 I=1,4
      DO 201 J=1,I
      PAD44I(J,I)=A(KK)
      PAD44I(I,J)=PAD44I(J,I)
  201 KK=KK+1
      CALL MPRD (PAD64,PAD44I,K,6,4,0,0,4)
C
C     UPDATE FILTER PREDICTION COVARIANCE MATRIX
C
      CALL MPRD (K,M,PAD66,6,4,0,0,6)
      DO 14 I=1,6
      DO 13 J=1,6
   13 PAD66(I,J)=-PAD66(I,J)
   14 PAD66(I,I)=1.0+PAD66(I,I)
      CALL MTRA (PAD66,PAD66T,6,6,0)
      CALL MPRD (PAD66,PP,PAD661,6,6,0,0,6)
      CALL MPRD (PAD661,PAD66T,PP,6,6,0,0,6)
      CALL MTRA (K,KT,6,4,0)
      DO 140 I=1,6
      DO 140 J=1,4
  140 PAD64(I,J)=K(I,J)*P(J)
      CALL MPRD (PAD64,KT,PAD66,6,4,0,0,6)
      DO 15 I=1,6
      DO 15 J=1,6
   15 PP(I,J)=PP(I,J)+PAD66(I,J)
C
C     CALCULATE OPTIMUM ESTIMATE OF ERRORS IN STATES
C
         Z(1) = RHOM-RHOC
      Z(2) = RHODM-RHODC
      IF( AC.LT.0.0)GO TO 100
      GO TO 300
 100  AC = AC + 360.0
 300  IF(C(001).GT.1.0)GO TO 50
      ACB = AC
      AMB = AM
 50   ACA = AC
      AMA = AM
C
C        CHECK FOR INCREASING OR DECREASING AZIMUTH
C
      IF(AZDOT) 18,17,17
 17   IF(AMA-AMB)20,10,10
 20   AMA = AMA + 360.0
  10  ACAB = ACA - ACB
      IF(ACAB.LT.-350.0)ACA = ACA+360.0
      GO TO 30
 18   IF(AMA-AMB)21,21,22
 22   AMA = AMA - 360.0
 21   IF(ACA-ACB)30,30,40
 40   ACA = ACA-360.0
 30   Z(3) = AMA-ACA
```

```
      Z(3) = Z(3)*CDTR
      ACR=ACA
      AMR = AMA
      Z(4) = EM-EC
      Z(4) = Z(4)*CDTR
      WRITE(6,NAME4)
C
      CALL MPRD(K,Z,DXEST,6,4,0,0,1)
C
C     UPDATE STATES
      DO 16 I=1,6
   16 X(I)=X(I)+DXEST(I)
      RETURN
      END

                  ******************************

$IBFTC SDM.
      SUBROUTINE SDM
C
C     THIS SUBROUTINE WHEN GIVEN THE CURRENT VALUES OF
C     POSITION AND VELOCITY COMPUTES THE SYSTEM
C     DESCRIPTION MATRIX TO BE USED IN SUBROUTINE UPCOV
C
      COMMON/AFIT/C(600)
      REAL MU,J2,J3
      EQUIVALENCE      (C(019),MU      ),(C(017),J2      ),
     1(C(018),J3     ),(C(091),X      ),(C(092),Y       ),
     2(C(093),Z      ),(C(204),F41    ),(C(210),F42     ),
     3(C(216),F43    ),(C(205),F51    ),(C(211),F52     ),
     4(C(217),F53    ),(C(206),F61    ),(C(212),F62     ),
     5(C(218),F63    ),(C(228),F45    ),(C(223),F54     ),
     6(C(012),AE     ),(C(014),WIE2   ),(C(013),WIE     )
      R=SQRT(X*X+Y*Y+Z*Z)
C
C     COMPUTE THE HARMONIC TERMS.
C
      A=MU/(R**3)
      AX=A*X
      AY=A*Y
      AZ=A*Z
      B=Z/R
      BB=B*B
      H=(J2*(AE**2))/(R**2)
      D=(J2*(AE**2))/(R**4)
      E=(J3*(AE**3))/(R**3)
      F=(J3*(AE**3))/(R**4)
      G = 15.0*F*Z
      GX = G*X
      GY = G*Y
      GXR = GX/(R**3)
      GYR = GY/(R**3)
```

```
      HA=1.5*H*(1.0-5.0*BB)+2.5*E*(3.0-7.0*BB)*B
      HB=1.5*H*(3.0-5.0*BB)+2.5*E*(6.0-7.0*BB)*B
      HA=HA+1.0
      HB=HB+1.0
      HAX=3.0*D*X*(10.0*BB-1.0)+GXR*(7.0*BB-2.0)
      HAY=3.0*D*Y*(10.0*BB-1.0)+GYR*(7.0*BB-2.0)
      HAZ=6.0*D*Z*(5.0*BB-3.0)+15.0*E*(0.5-(11.0/2.0)*BB+
     17.0*(BB**2))
C
C     COMPUTE THE STATE DESCRIPTION MATRIX F.
C
      F41=A*((3.0*((Y/R)**2)-1.0)*HA-X*HAX)+WIE2
      F42=AY*((3.0*Y/(R**2))*HA-HAY)
      F43=AX*((3.0*Z/(R**2))*HA-HAZ)
      F45=2.0*WIE
      F51=AY*((3.0*X/(R**2))*HA-HAX)
      F52=A*((3.0*((Y/R)**2)-1.0)*HA-Y*HAY)+WIE2
      F53=AY*((3.0*Z/(R**2))*HA-HAZ)
      F54=-2.0*WIE
      HBX=X*D*(30.0*BB-9.0)+GXR*(7.0*BB-4.0)
      HBY=Y*D*(20.0*BB-9.0)+GYR*(7.0*BB-4.0)
      HBZ=Z*6.0*D*(5.0*BB-4.0)+15.0*E*(1.0-7.5*BB+7.0*
     1(BB**2))
      F61=AZ*((3.0*X/(R**2))*HB-HBX)
      F62=AZ*((3.0*Y/(R**2))*HB-HBY)
      F63=A*((3.0*BB-1.0)*HB-Z*HBZ)
      RETURN
      END
```

```
$DATA
 $NAME5
      DT = 0.1
 $END
 $NAME1
 EP=0.06, AB=0.05, R3=-800.0, RDP=0.0
 $END
 $NAME2
 P11=1.0E02,P22=1.0E02,P33=1.0E02,P44=1.0E01,P55=1.0E01,P66=1.0E01
 $END
 $NAME3
 R1=1000.0, R2=2.0, R3=0.06, R4=0.05
 $END
 $NAME6
 NR=11.0,PINUT=47.0,SEC=12.782,NO=26,TI=10.1
 $END
    +765682+2917139000370
U03826348690781043122261115384 0736084 90678551 -641981  1511 -2501   2033 34D3462

U03826348690781043223581129803 0707602 84289151 -618763  1456 -2969   2553 34D4462

U03826348690781043324901145432 0673984 78177551 -589554  1607 -3521   3210 34D4462

U03826348590781043426161162370 0634048 72459501 -552659  1703 -4072   4047 34D4462

U03826348690781043527491180011 0586725 67158151 -506110  1758 -4804   5130 34D4462

U03826348690781044028821198495 0531024 62481851 -447467  1840 -5733   6441 34D4462

U03826348690781044130097216705 0464035 58476901 -374377  1730 -6532   7257 34D4462

U03826348690781044231392232855 0386362 55421101 -285620  1414 -7541   9540 34D4462

U03826348590781044332732245215 0298801 53221851 -191807   961 -0210  10897 34D4462

$EOF
```

Appendix E

Additional figures for Groups I and II are presented in this appendix. Figures 58-74 are from Group I results, and Figures 75-98 are from Group II results.

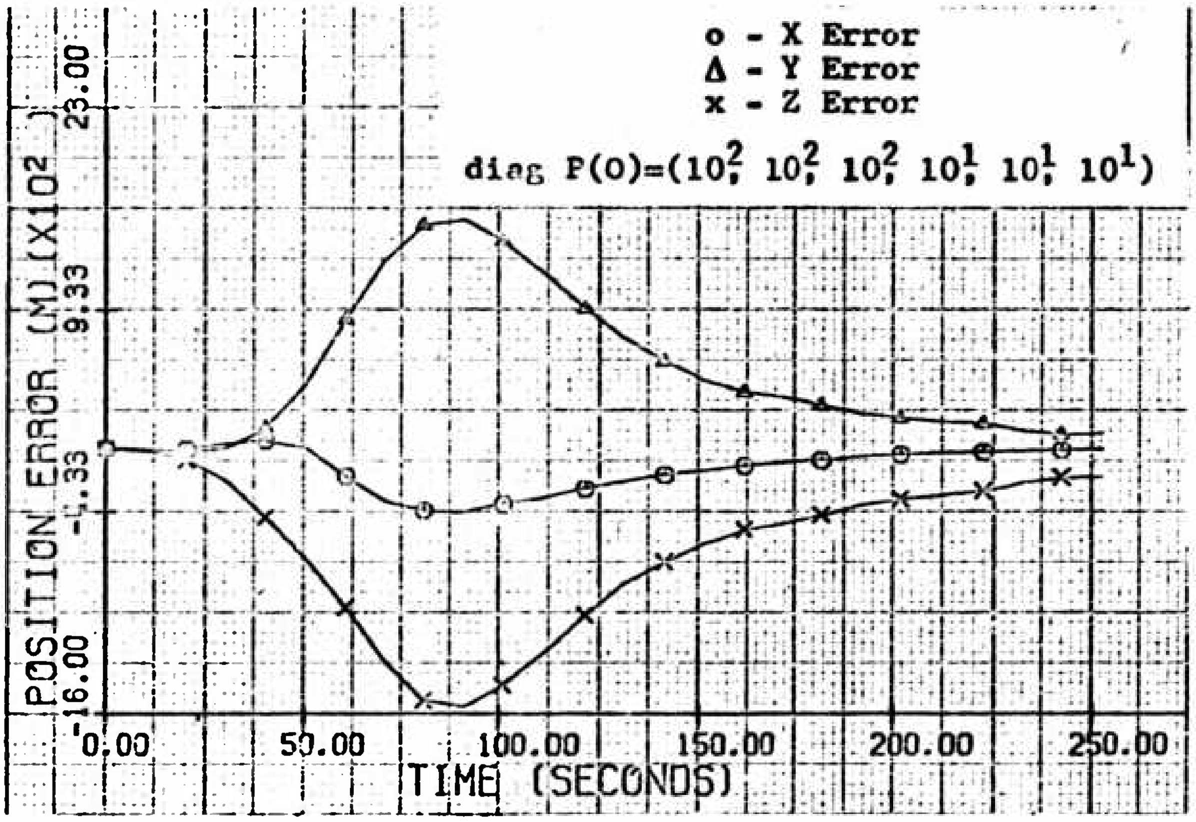


Fig. 57 Satellite 3826 Station 348 Pass #2

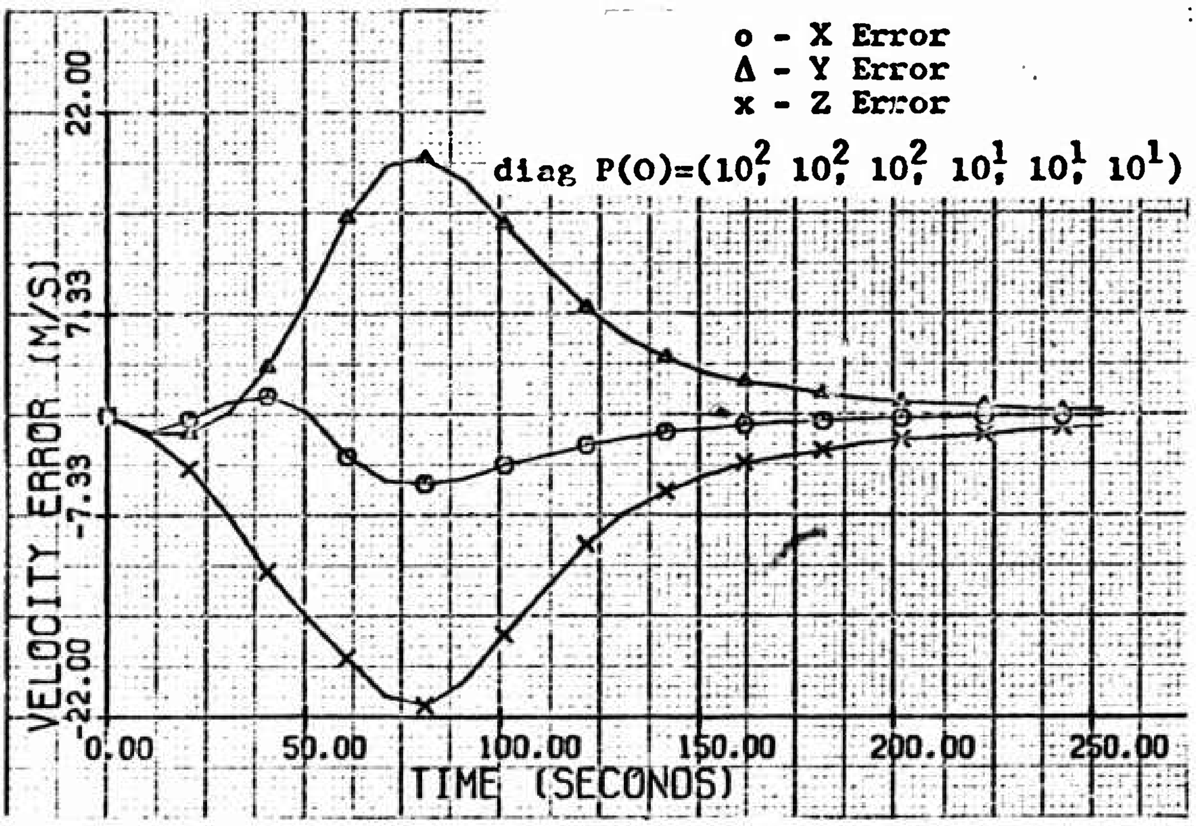


Fig. 58 Satellite 3826 Station 348 Pass #2

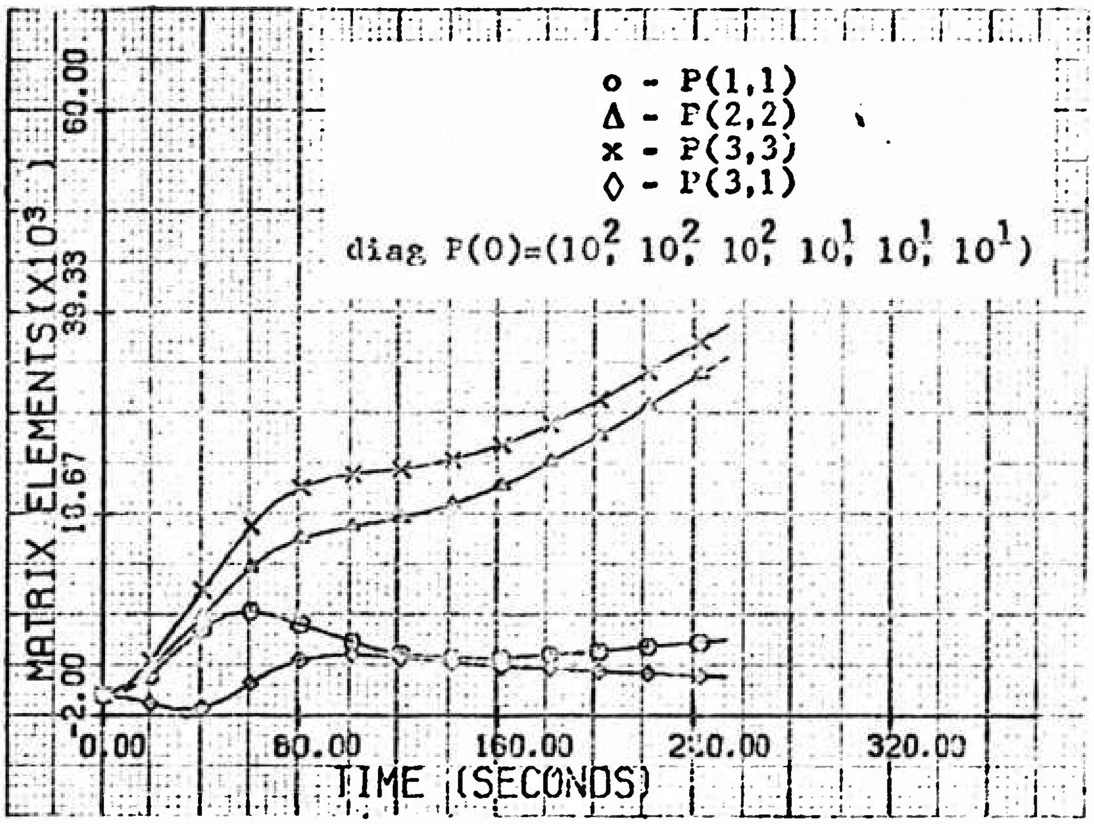


Fig. 59 Satellite 3826 Station 348 Pass #2

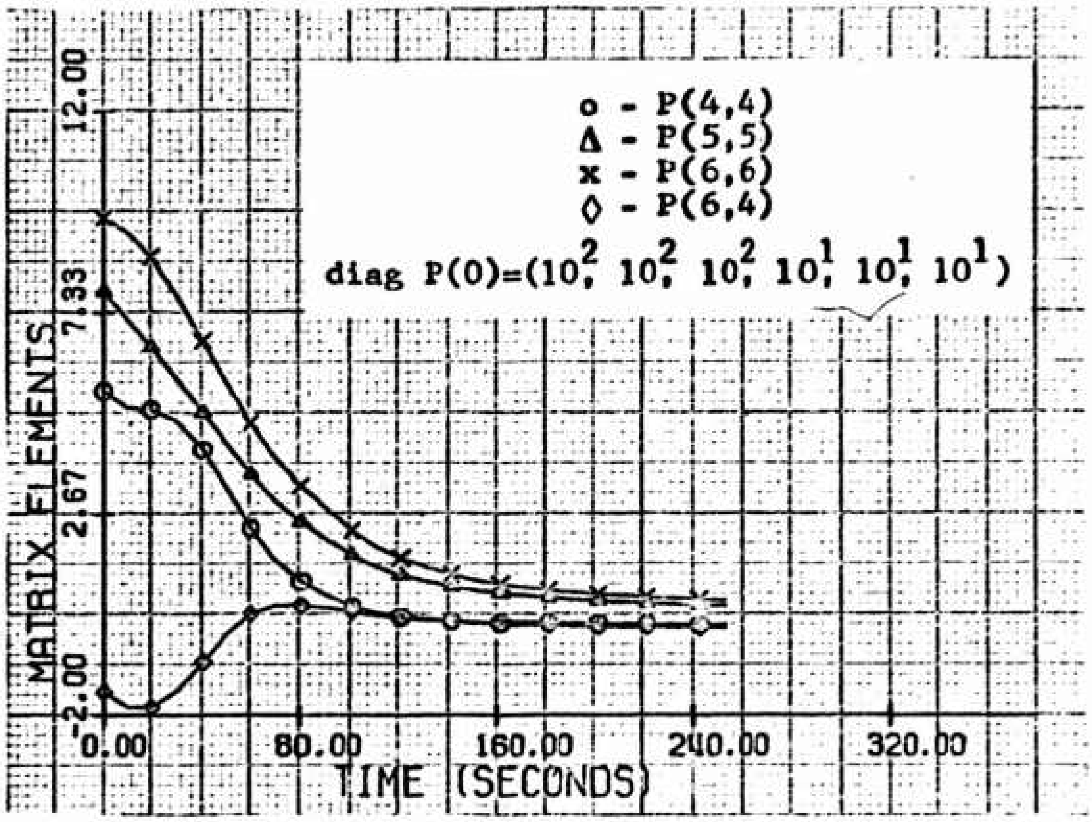


Fig. 60 Satellite 3826 Station 348 Pass #2

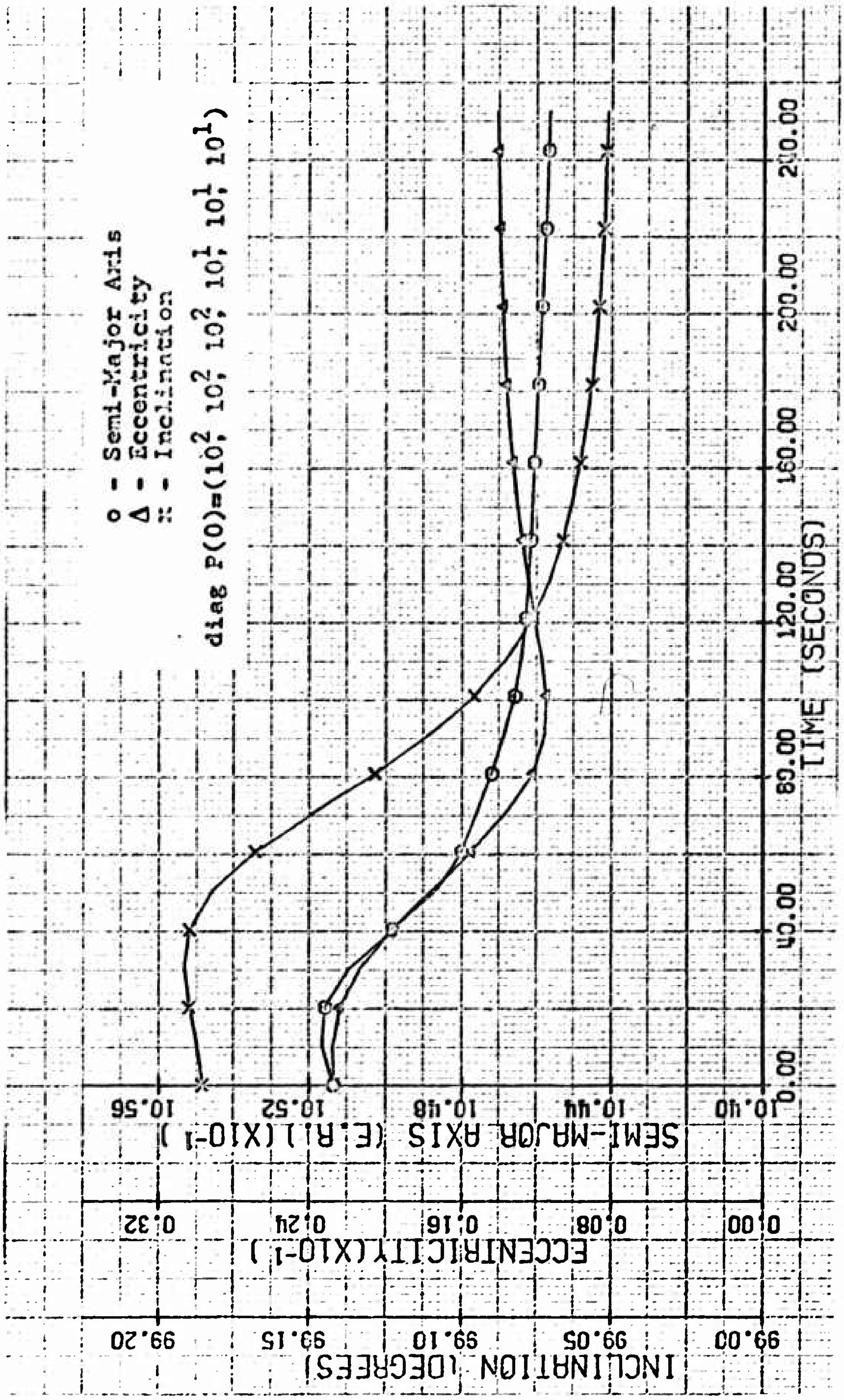


Fig. 61 Satellite 3826 Station 348 Pass #2

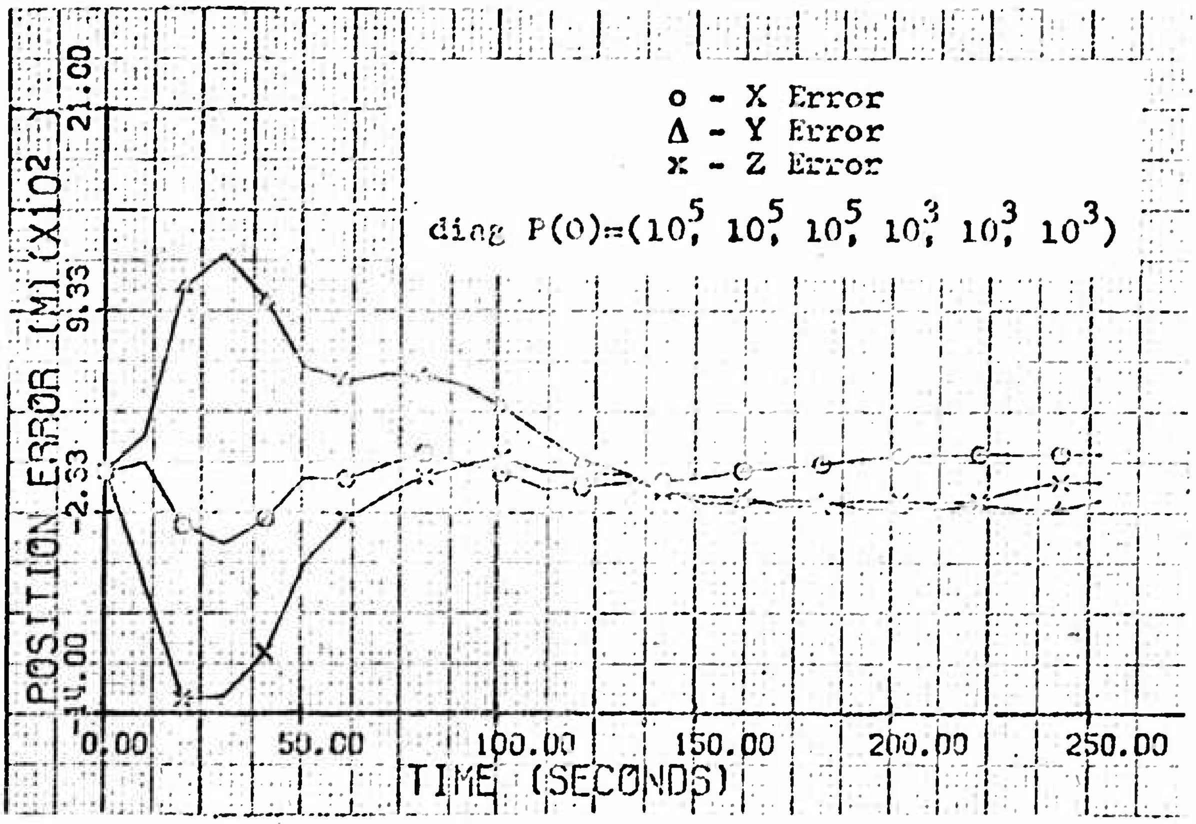


Fig. 62 Satellite 3826 Station 348 Pass #2

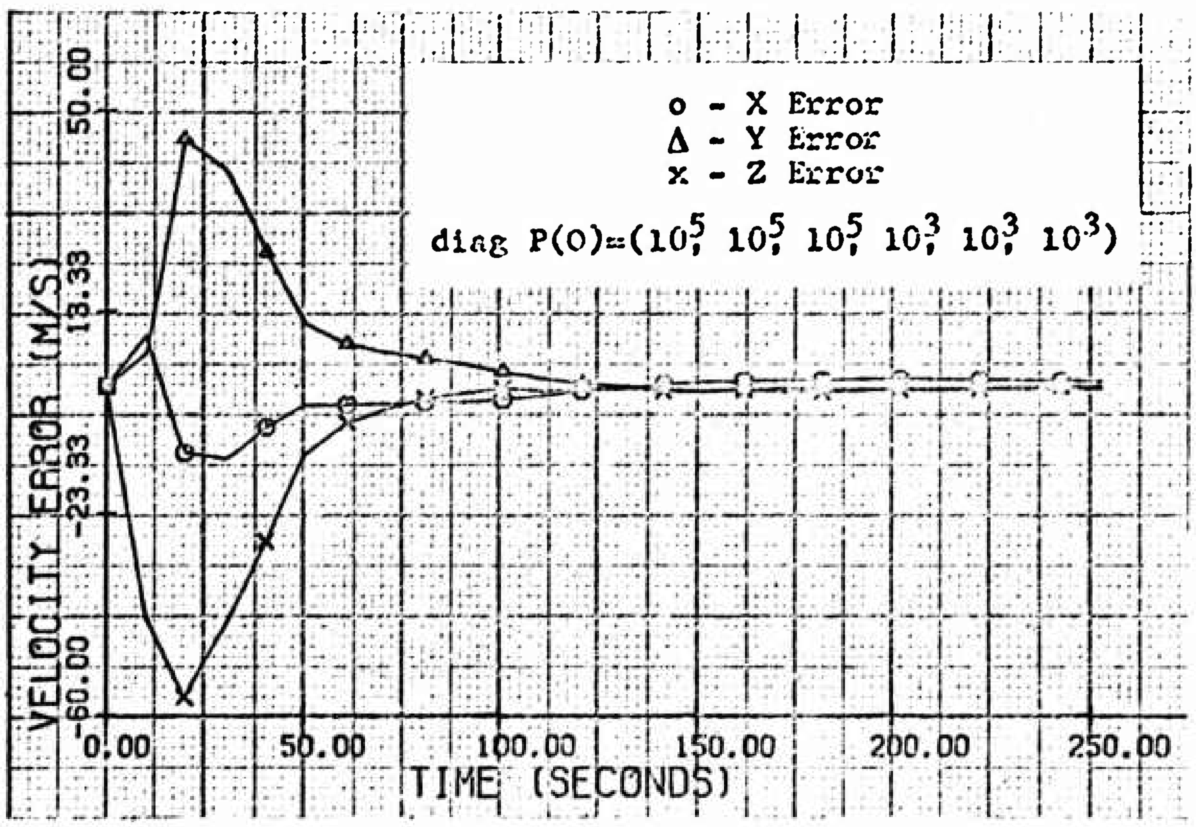


Fig. 63 Satellite 3826 Station 34? Pass #2

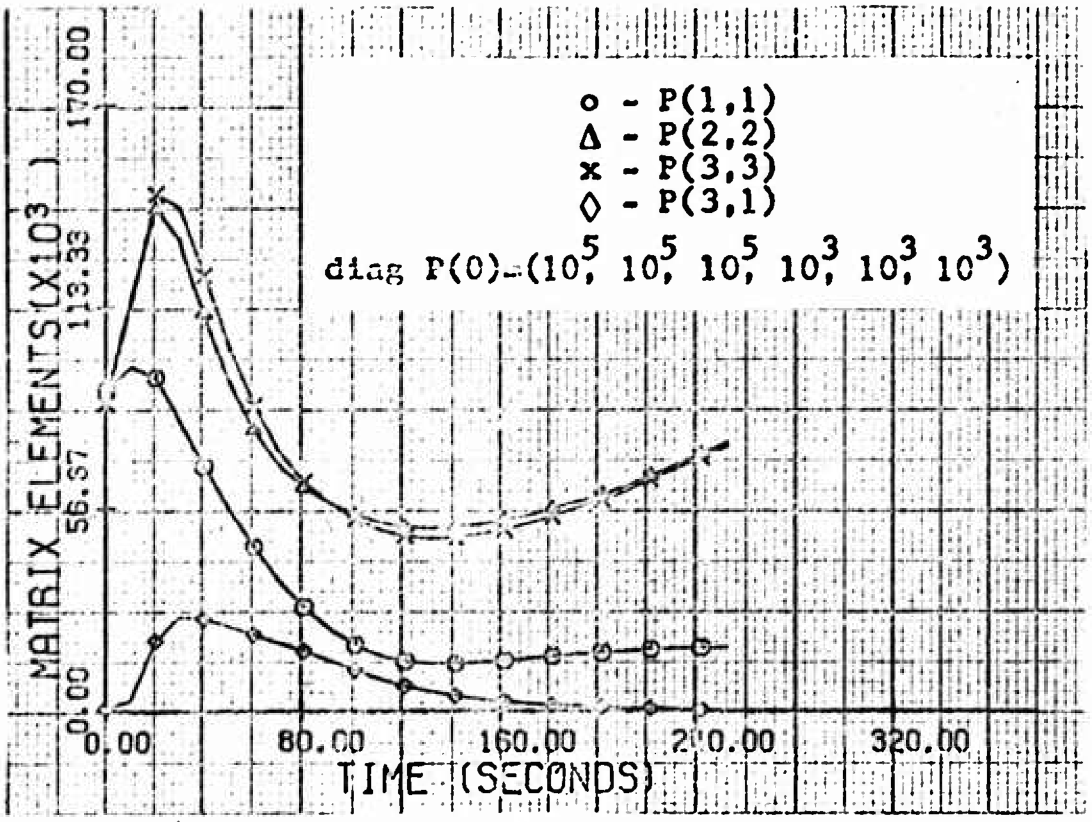


Fig. 64 Satellite 3826 Station 348 Pass #2

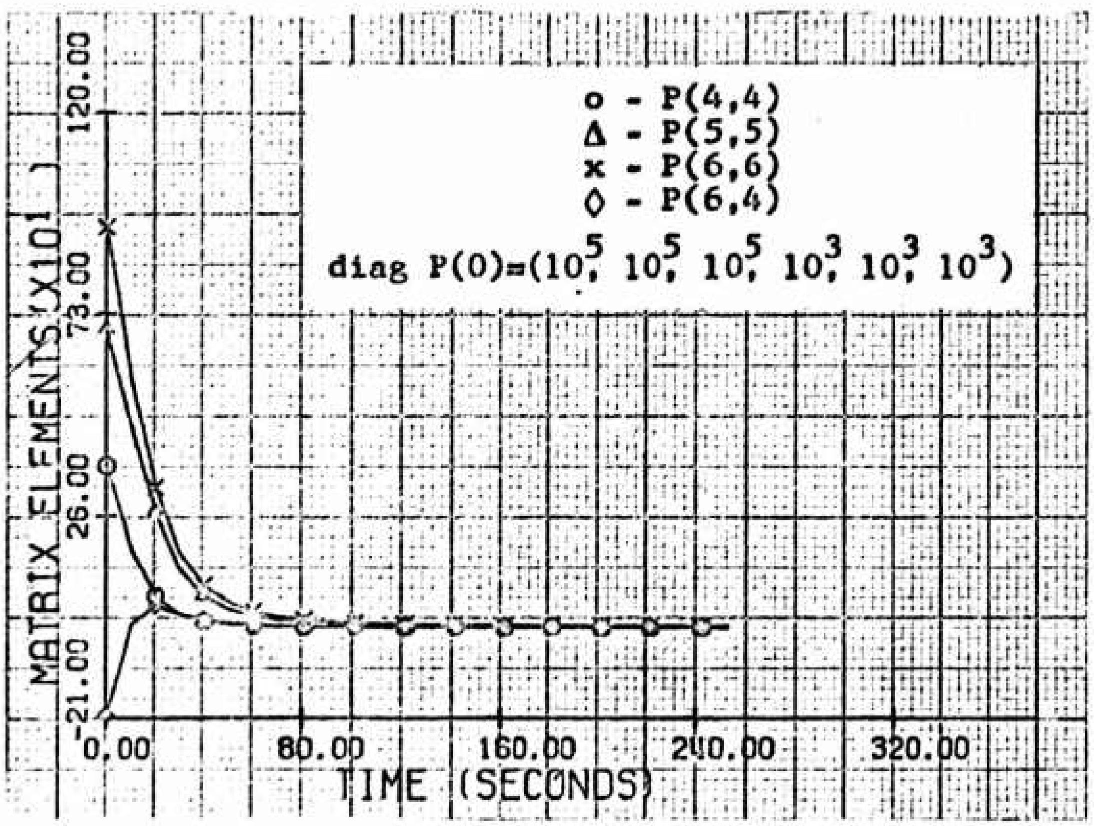


Fig. 65 Satellite 3826 Station 348 Pass #2

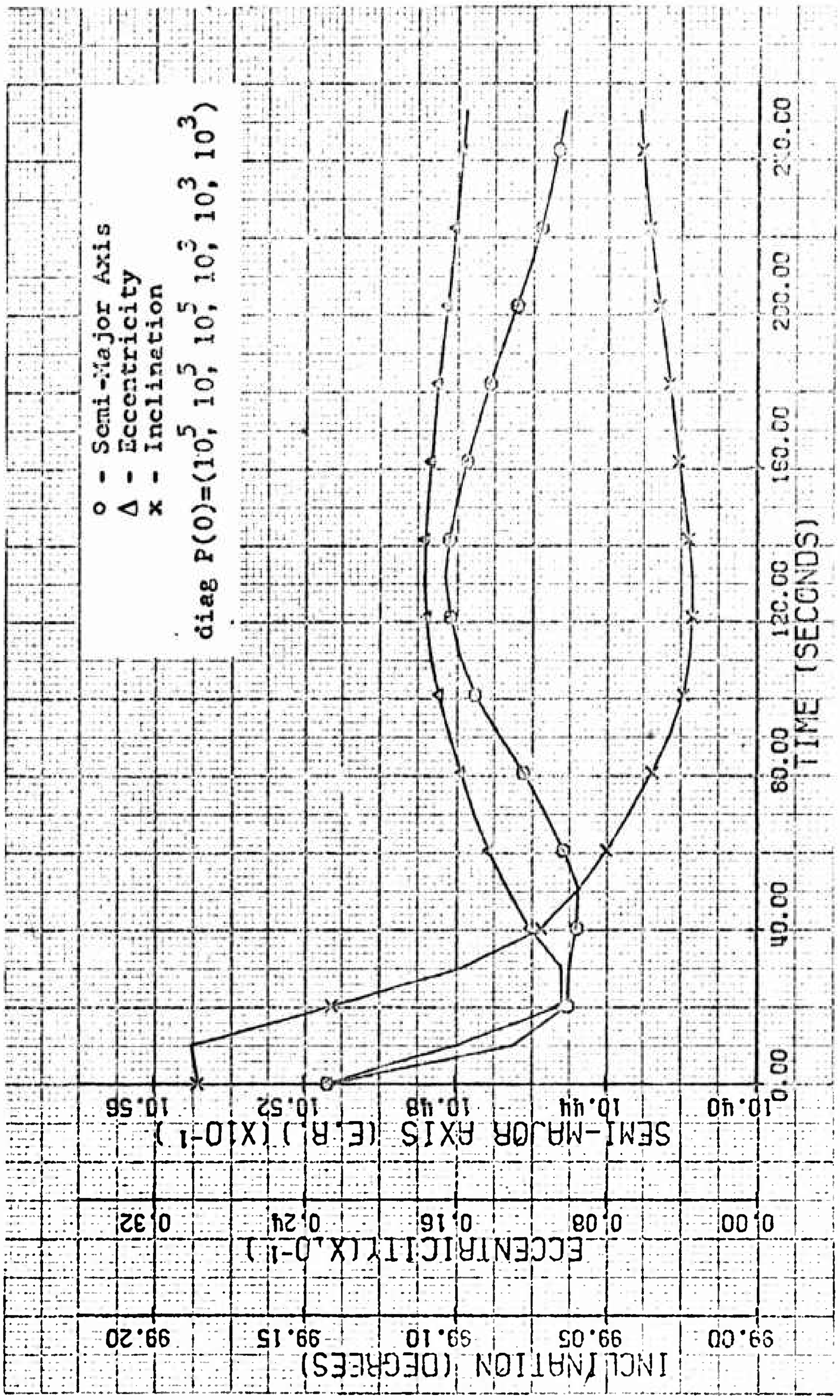


Fig. 66 Satellite 3826 Station 348 Pass #2

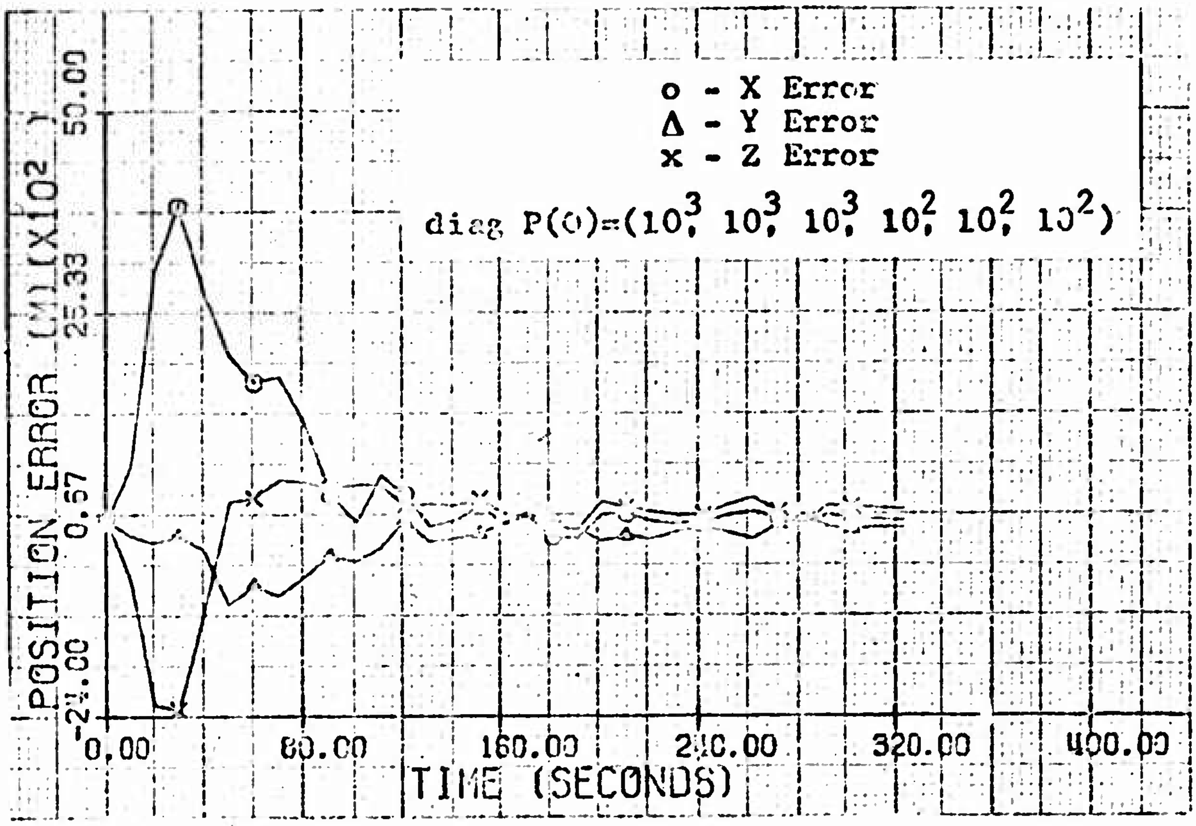


Fig. 67 Satellite 3824 Station 348 Pass #1

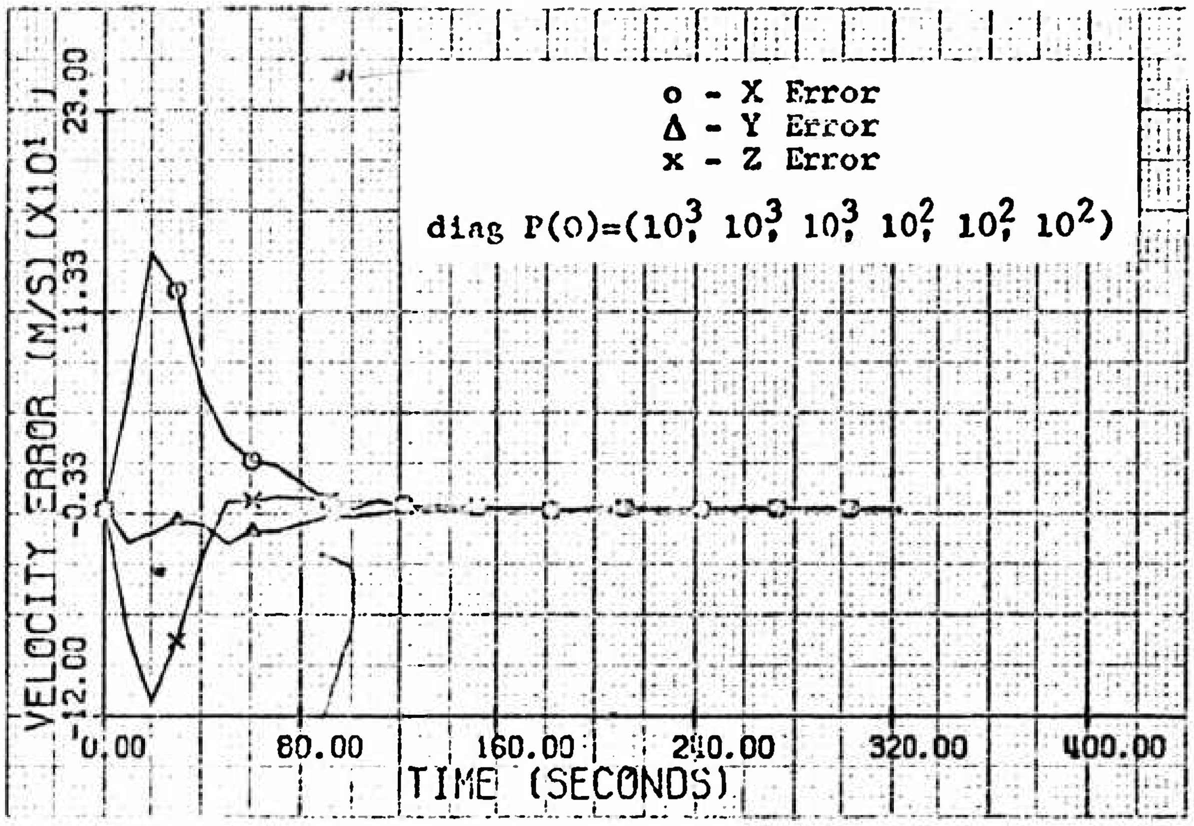


Fig. 68 Satellite 3824 Station 348 Pass #1

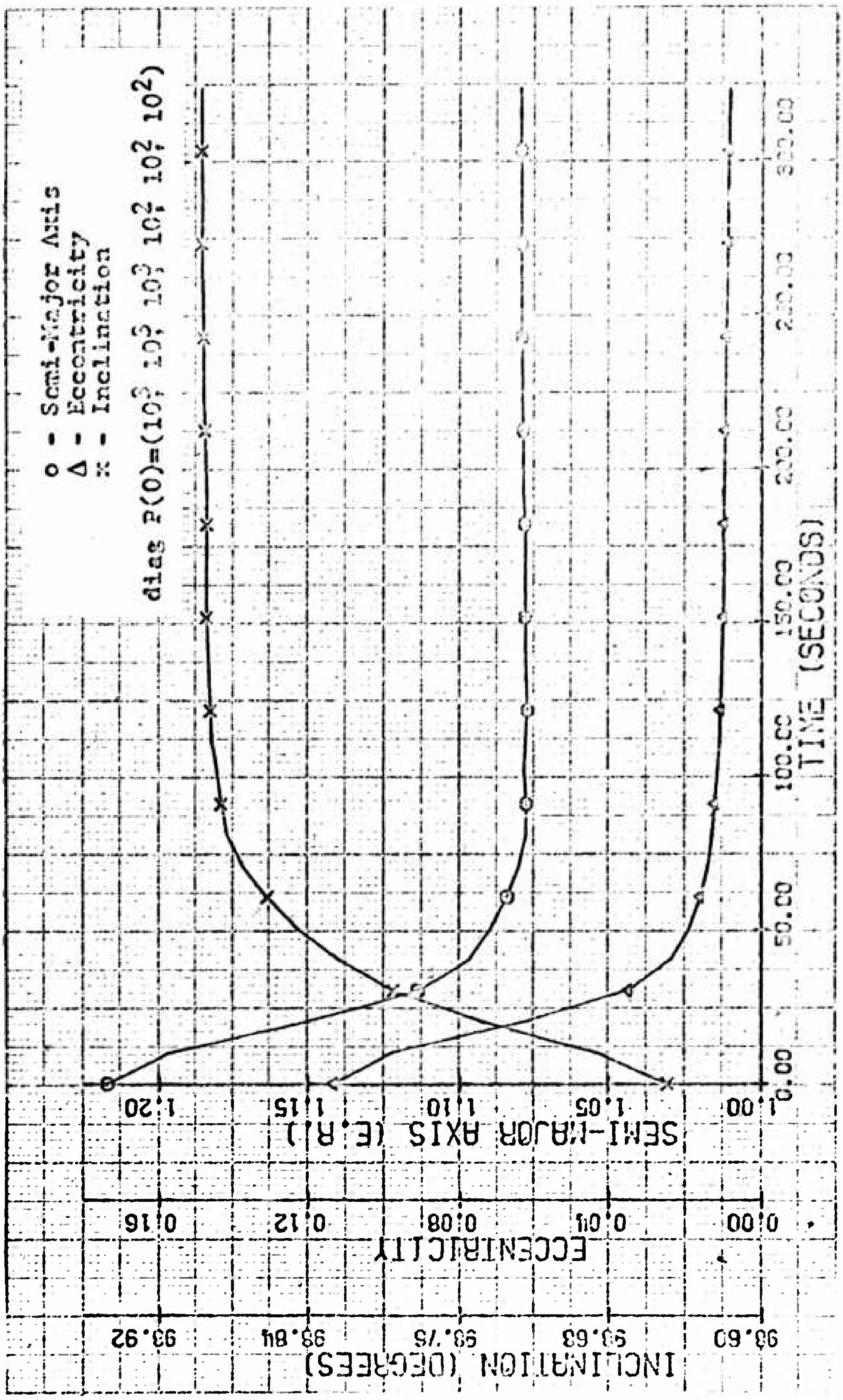


Fig. 69 Satellite 3824 Station 348 Pass #1

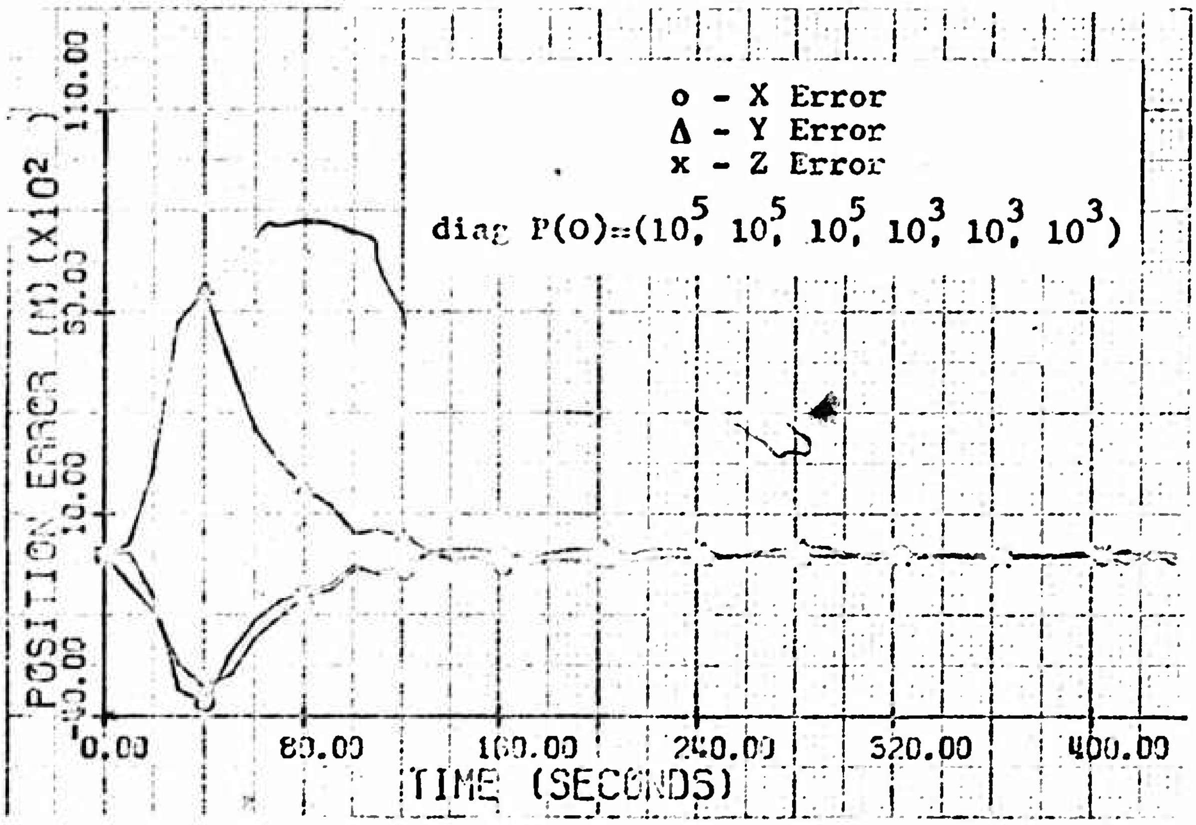


Fig. 70 Satellite 3823 Station 348 Pass #8

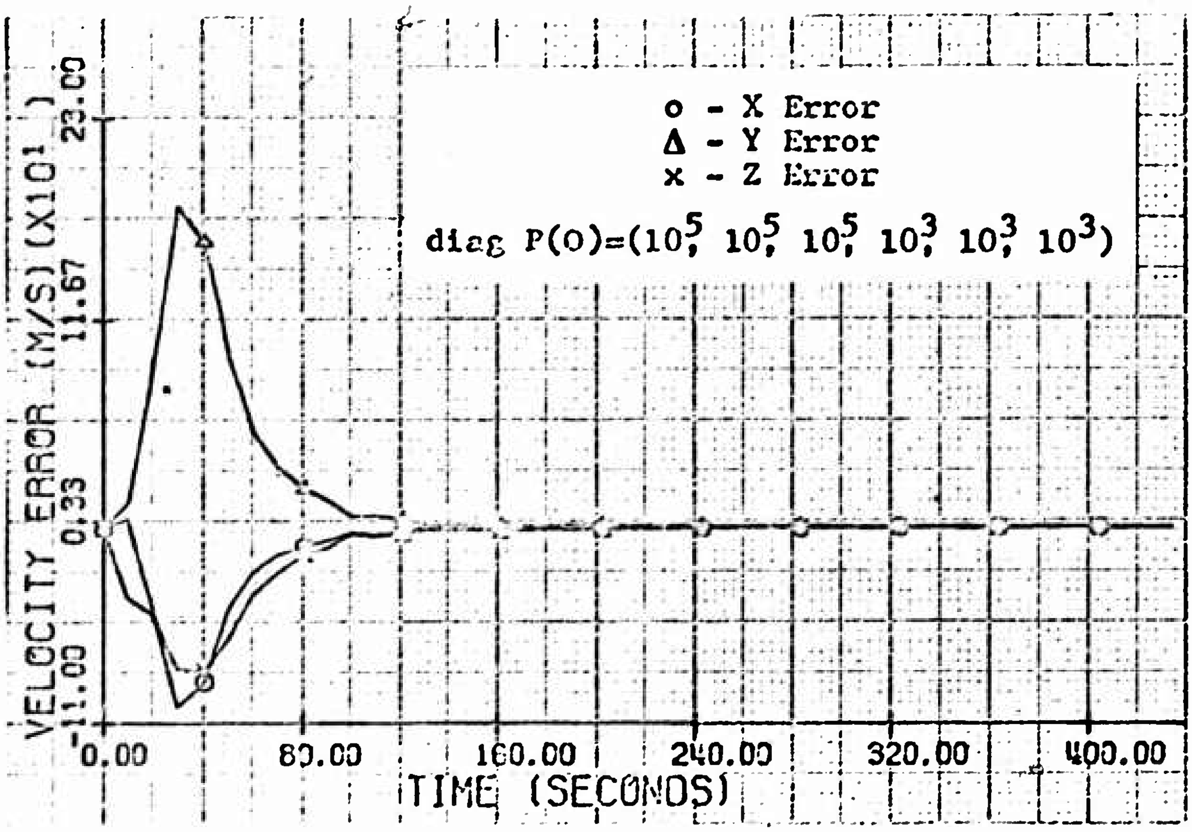


Fig. 71 Satellite 3823 Station 348 Pass #8

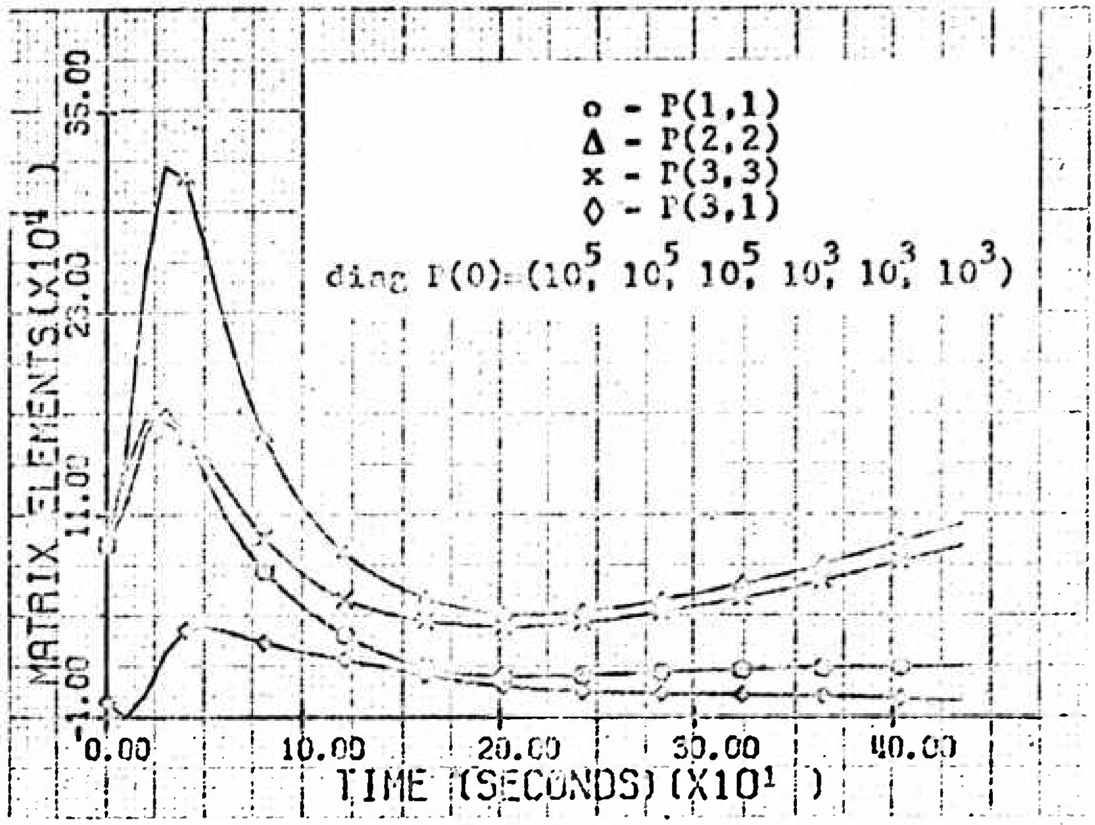
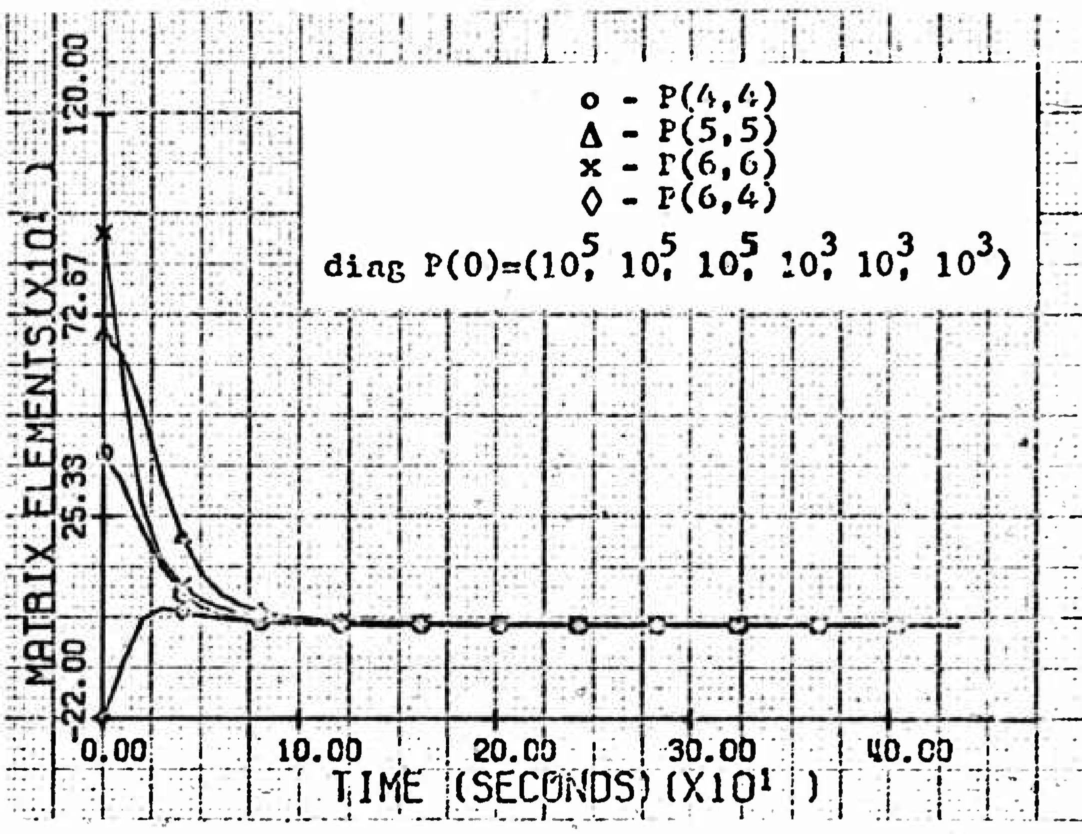


Fig. 72 Satellite 3823 Station 348 Pass #8



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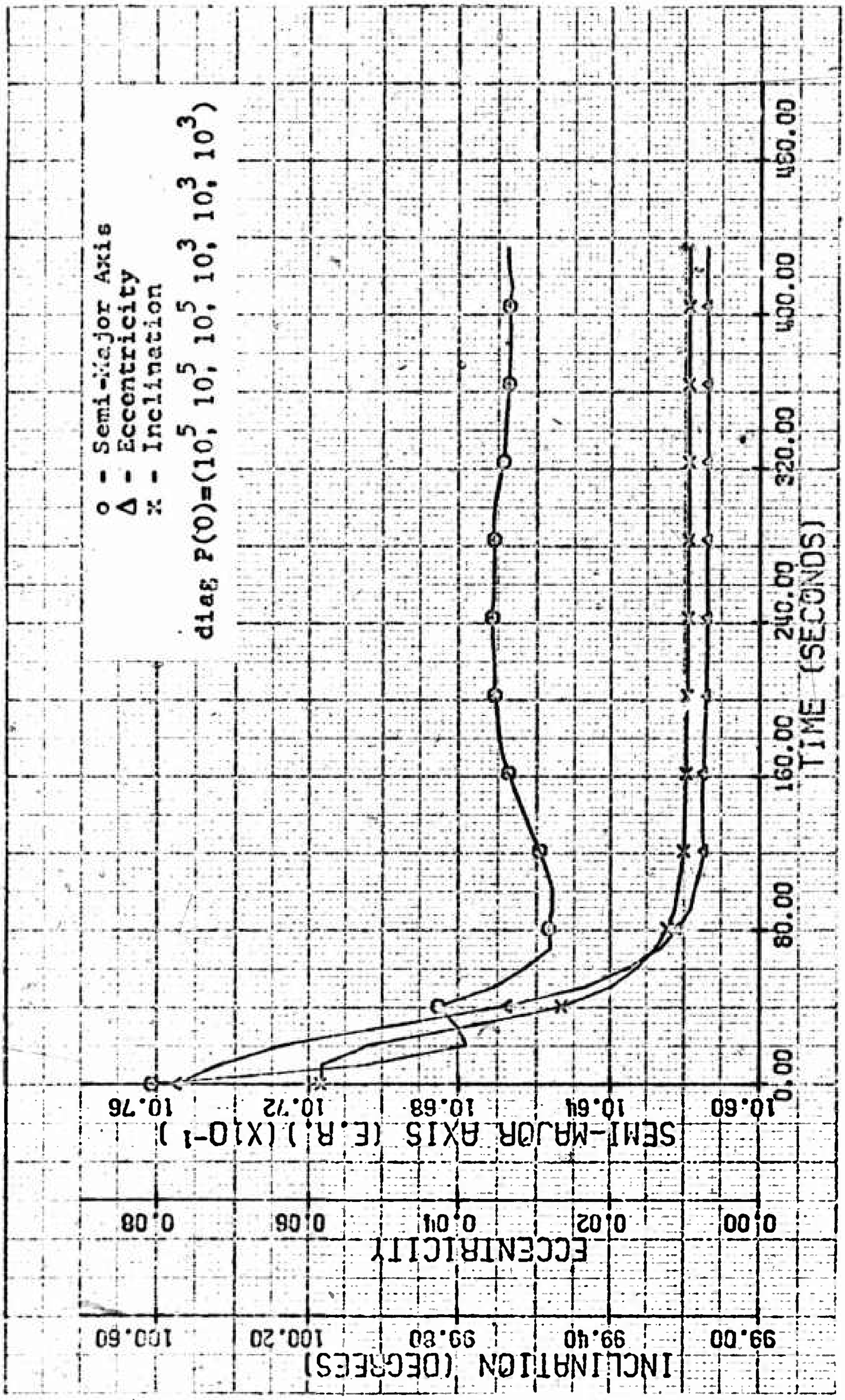

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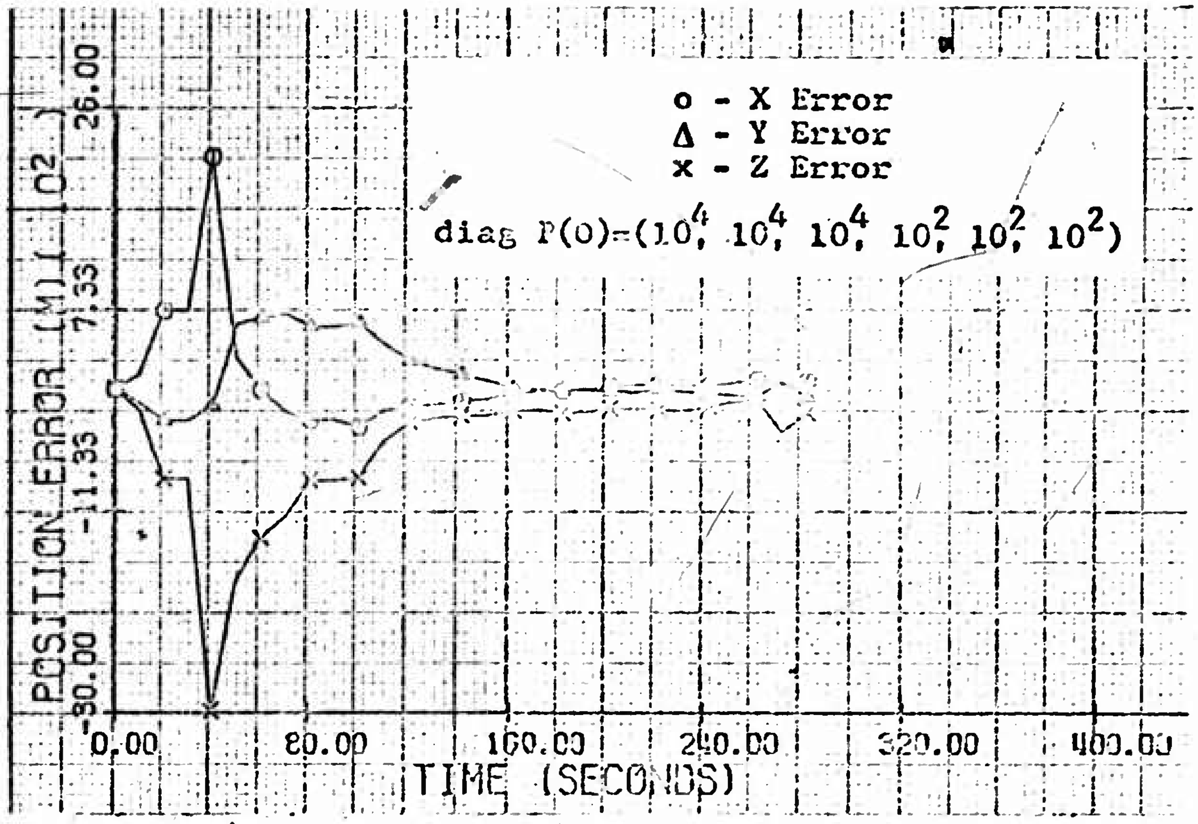

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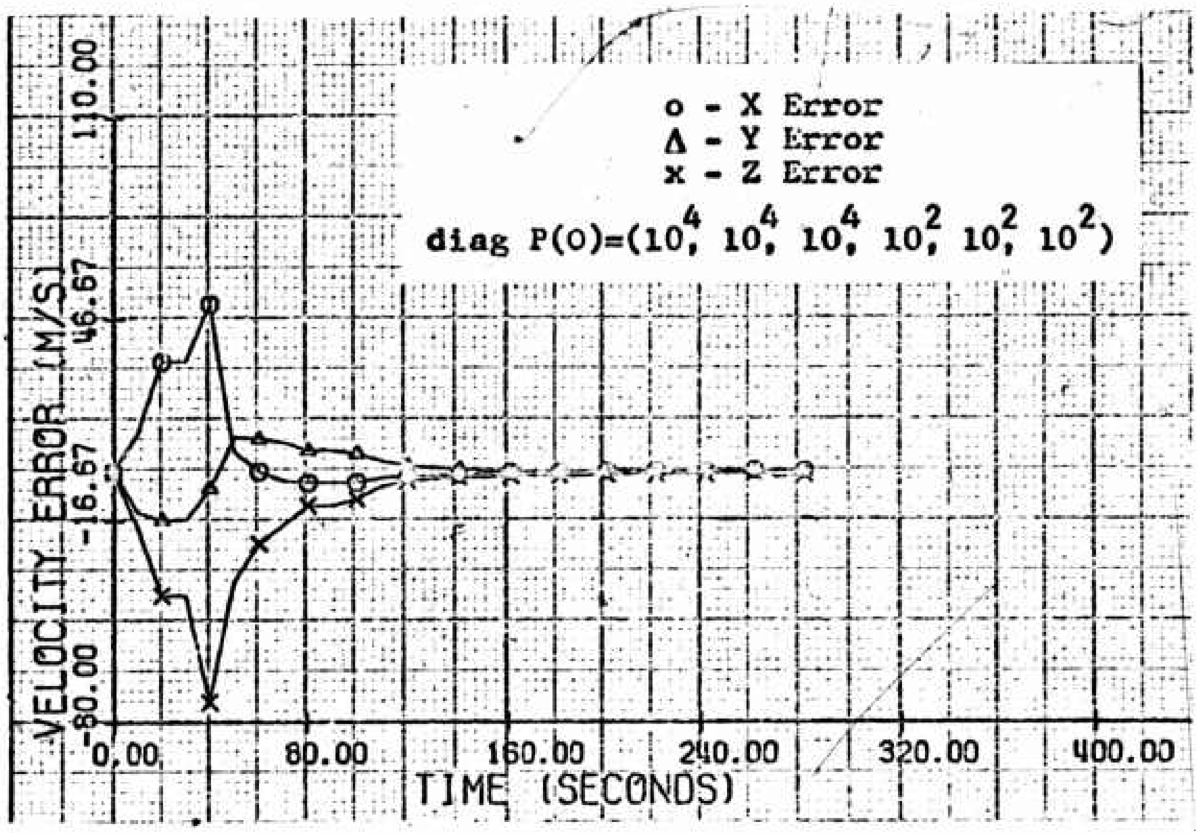

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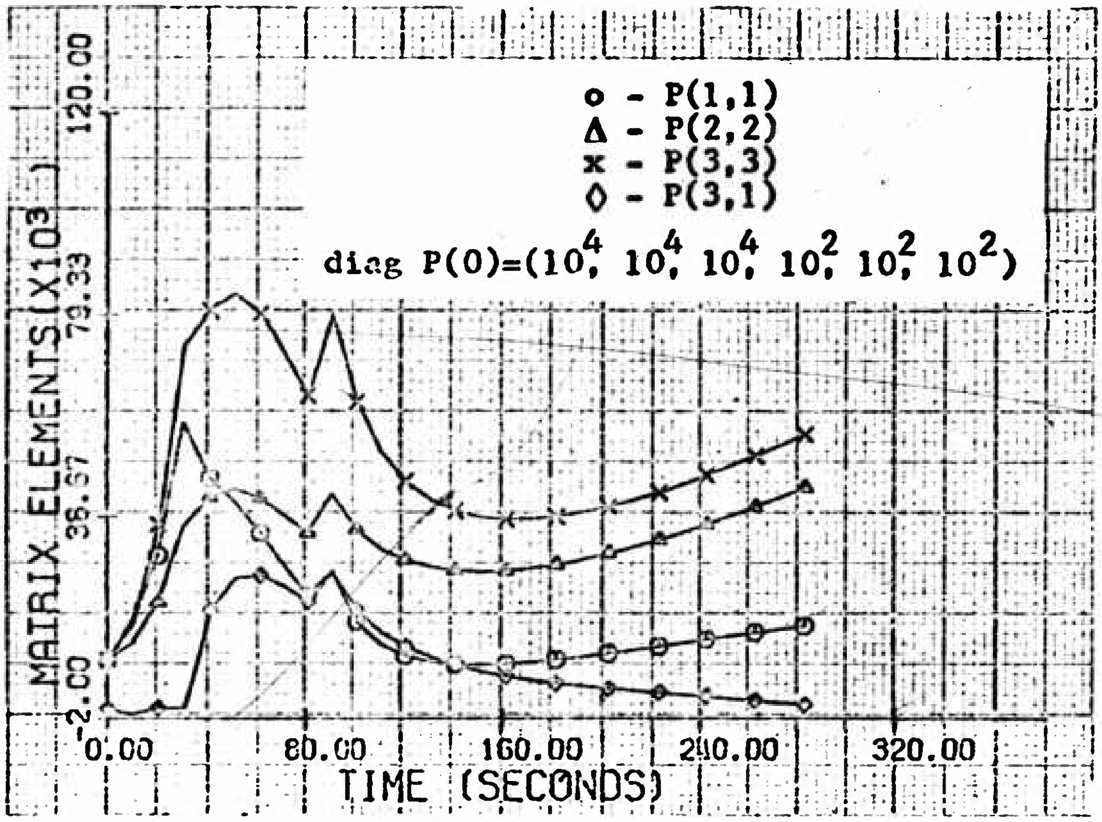

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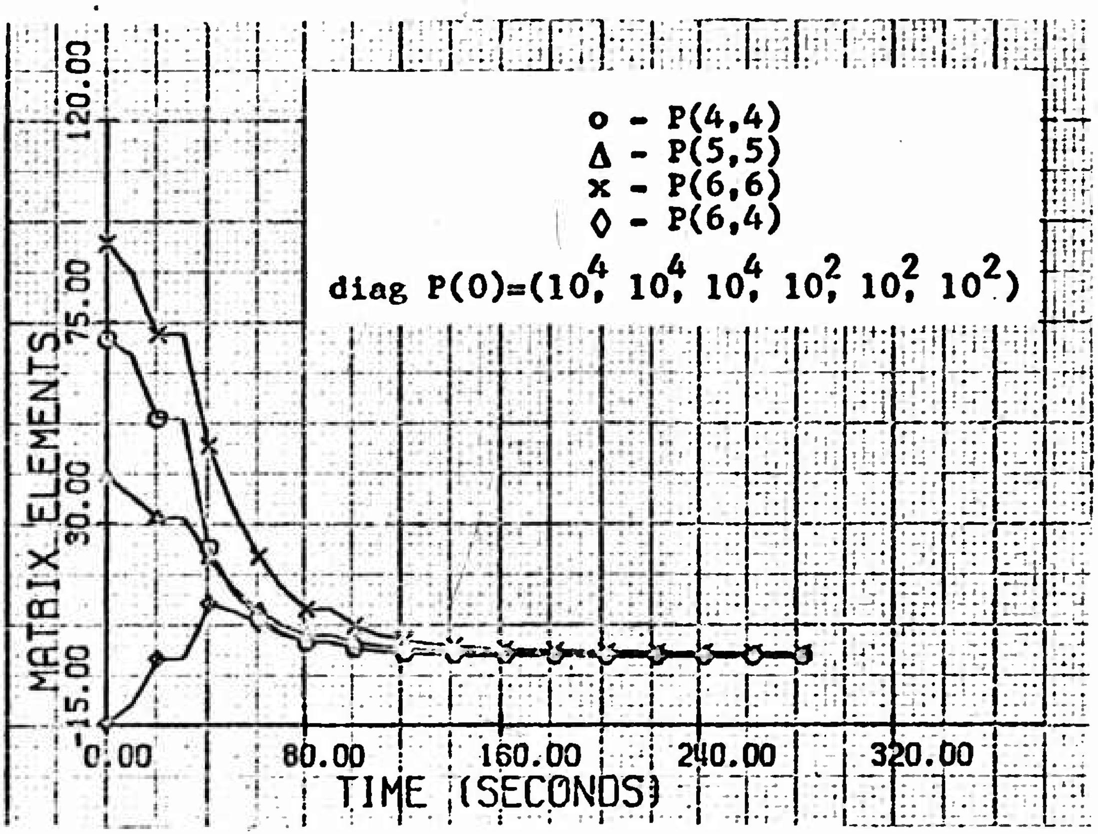

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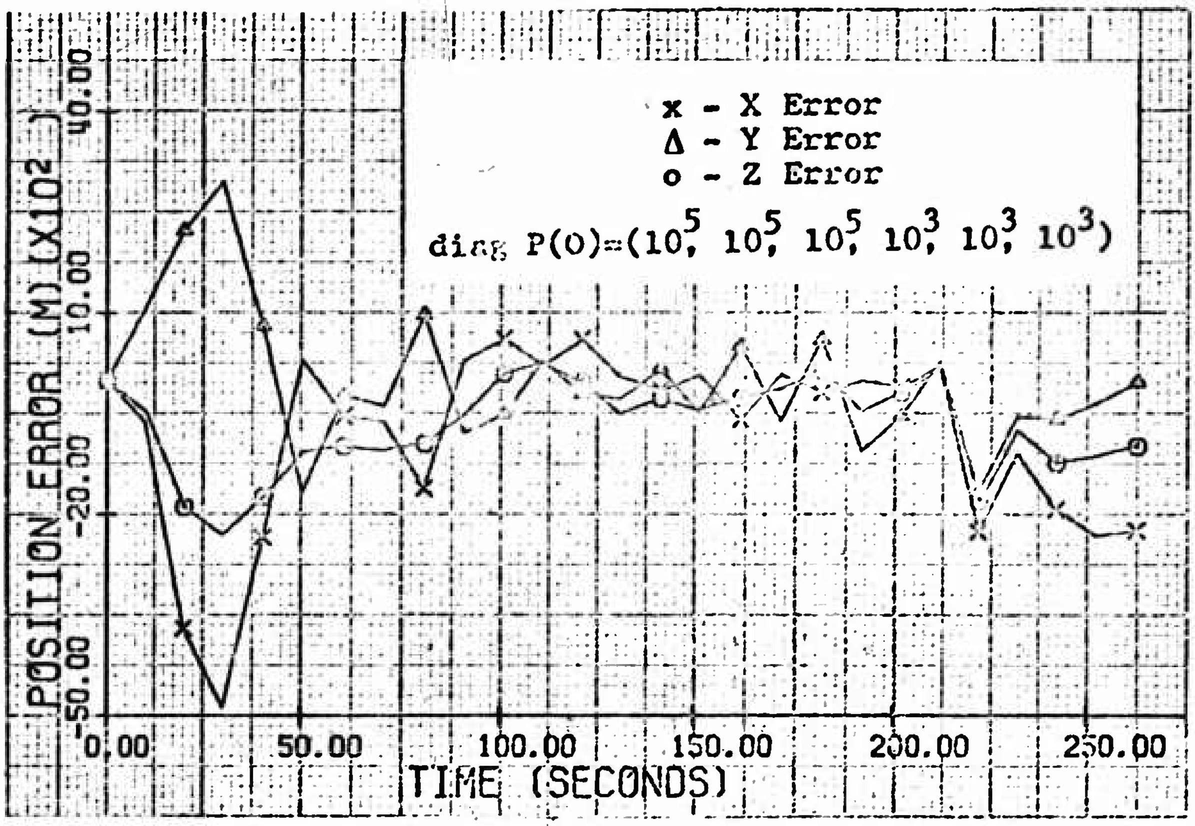

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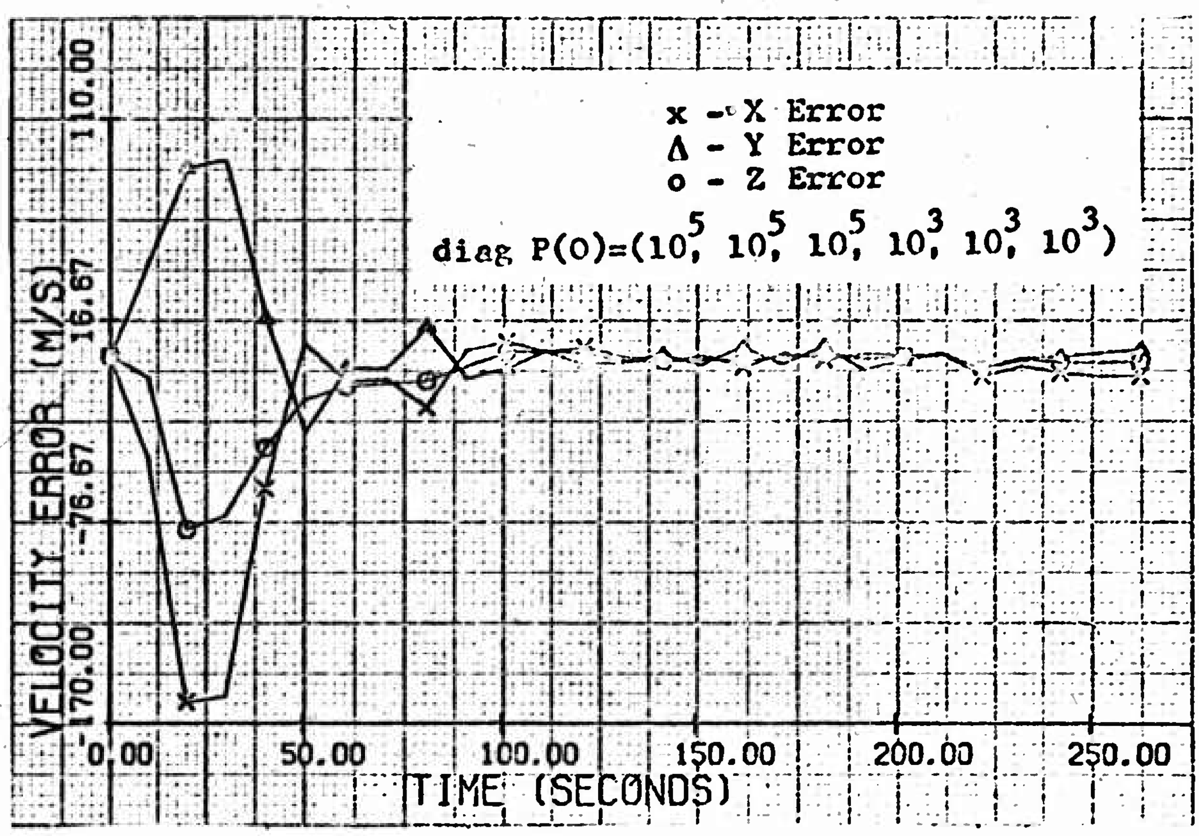

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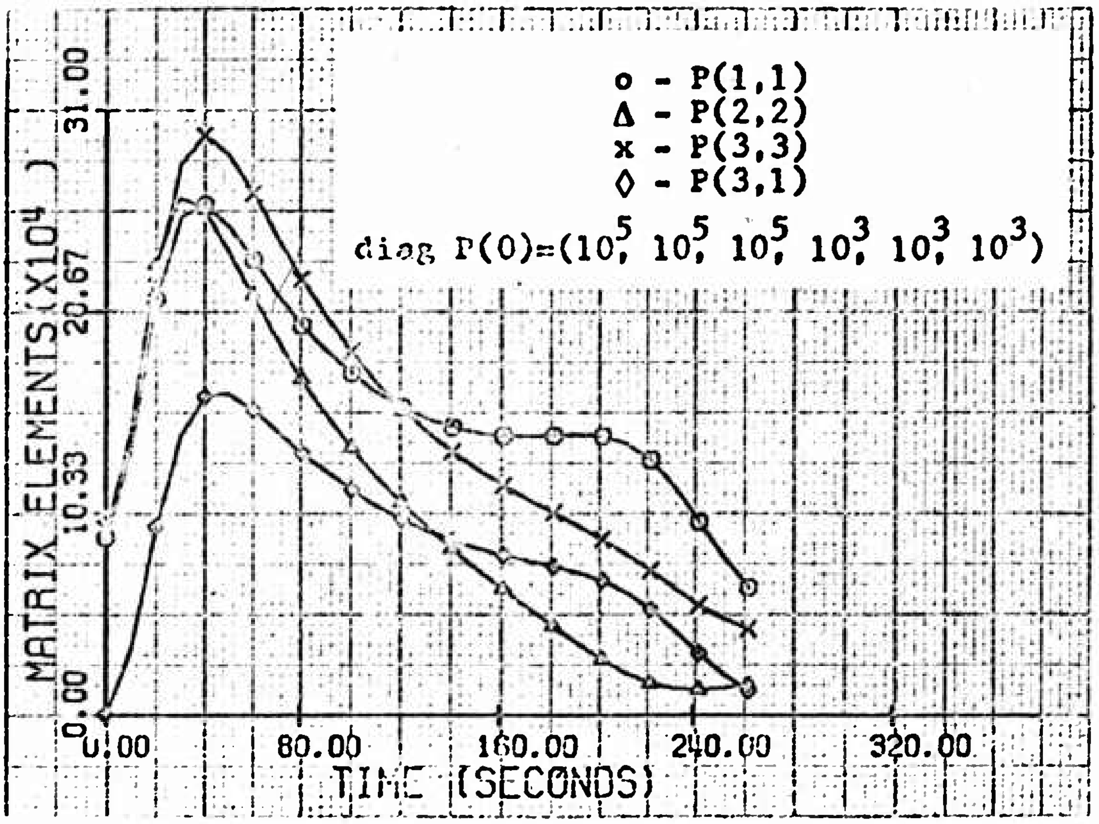

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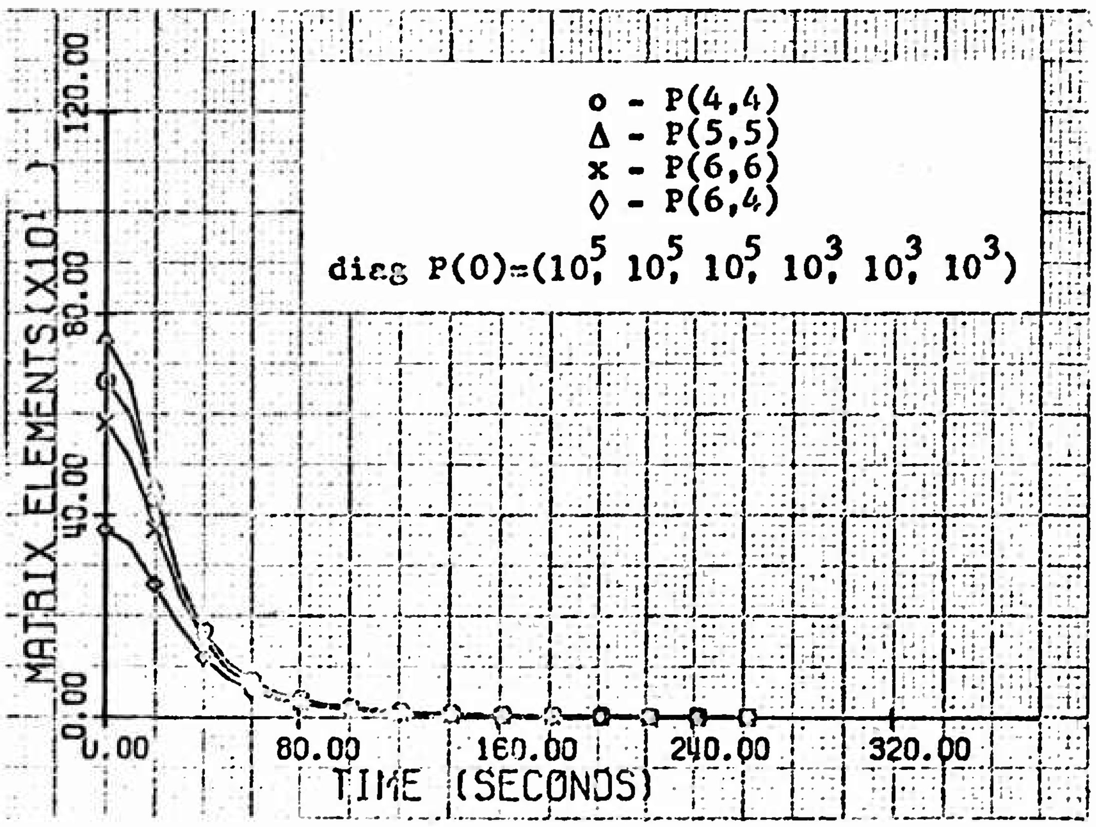

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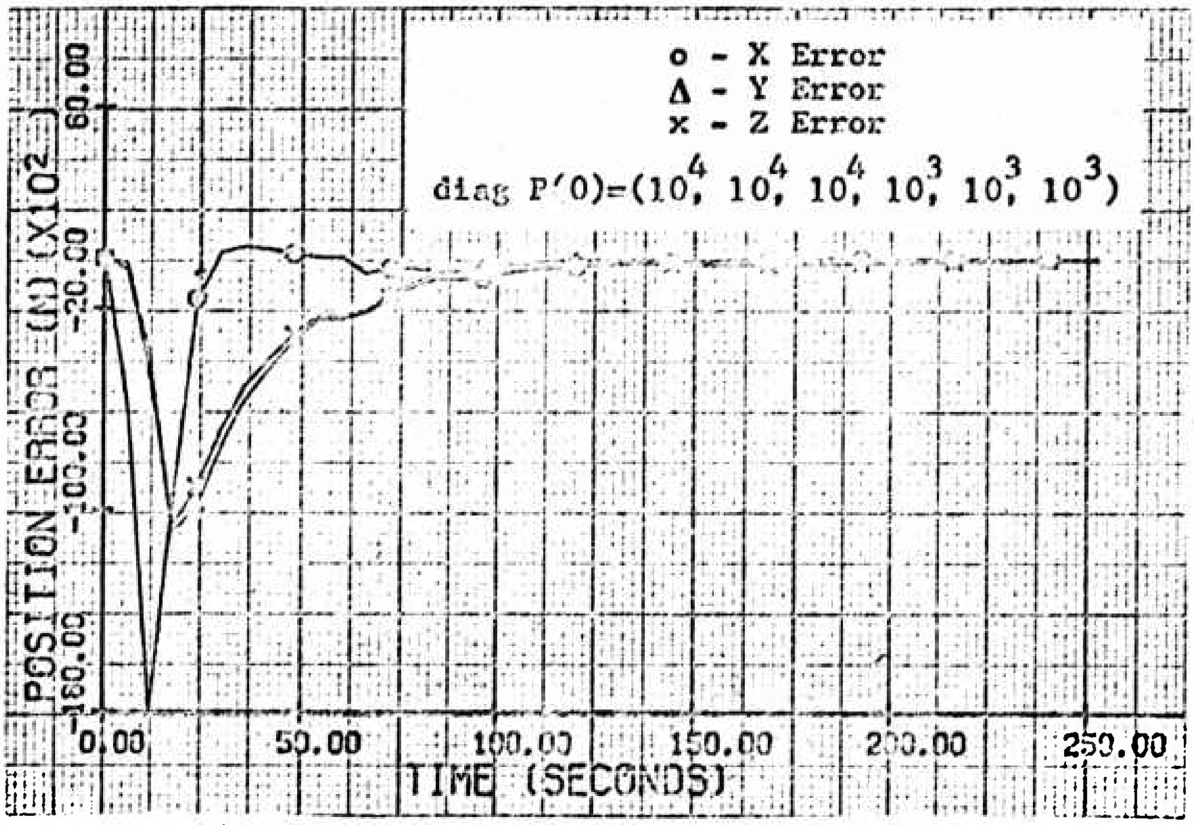

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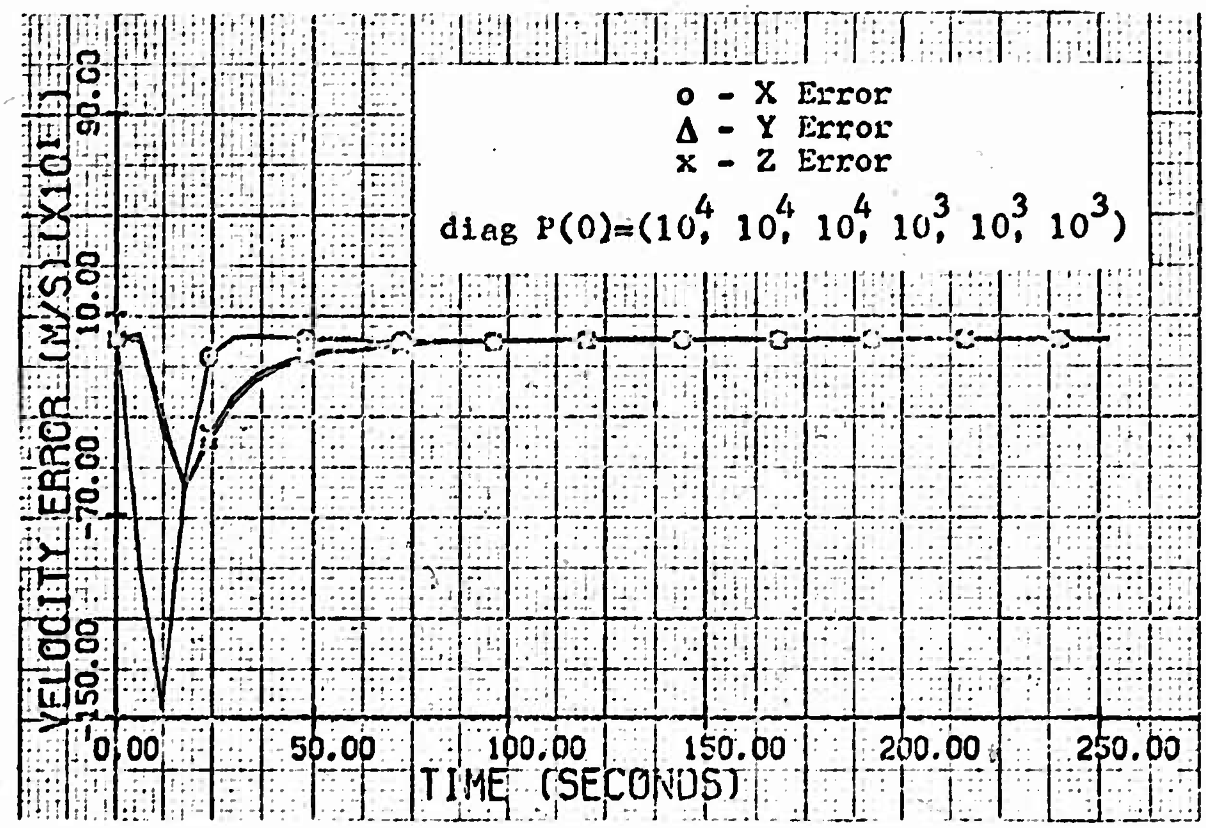

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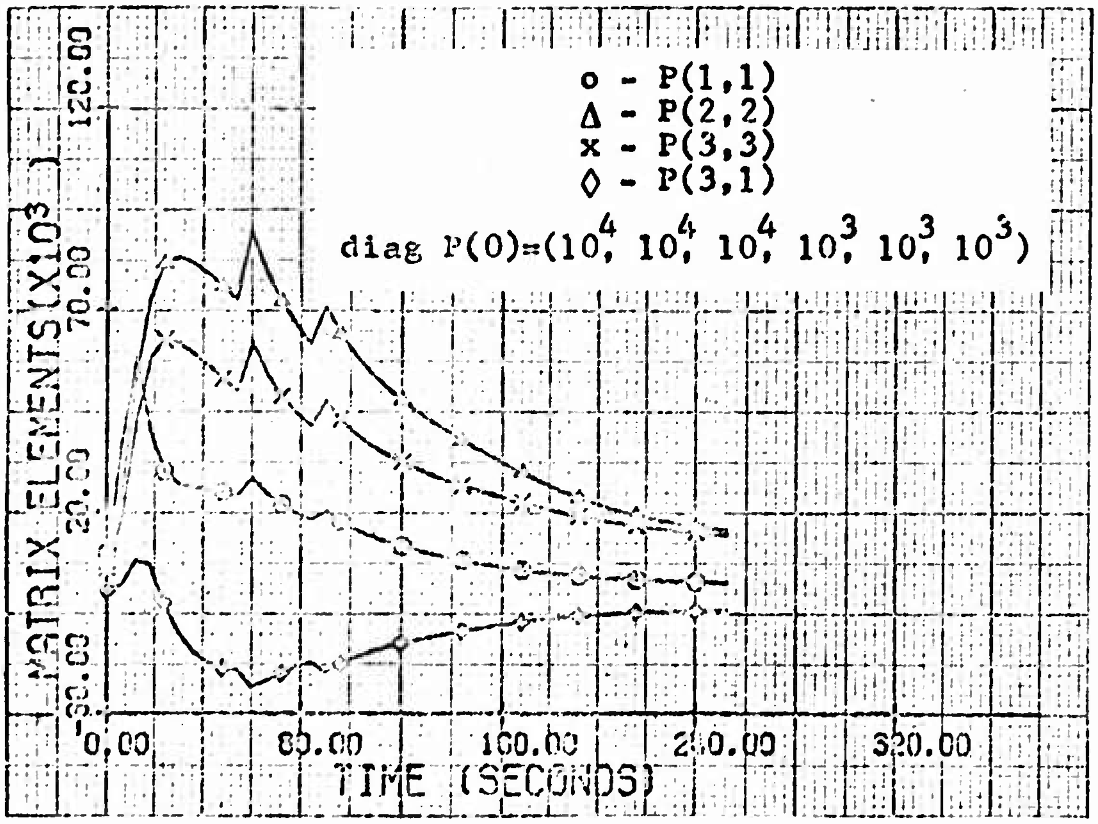

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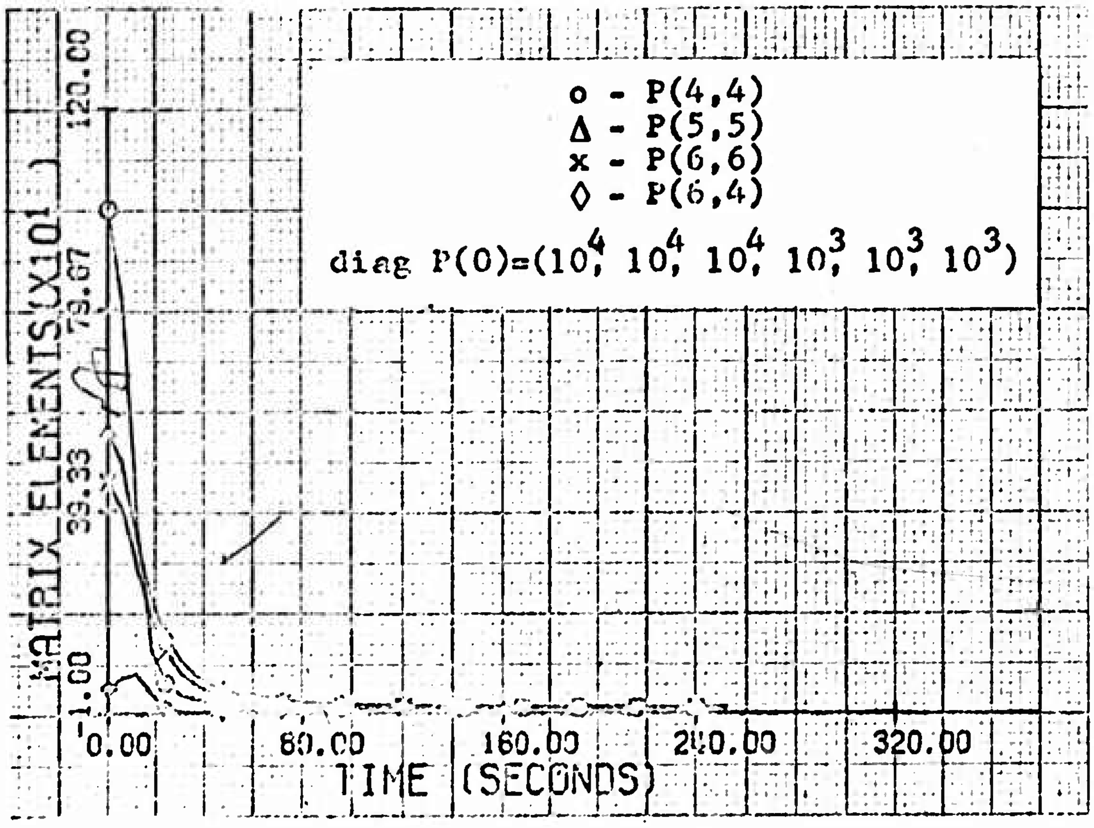

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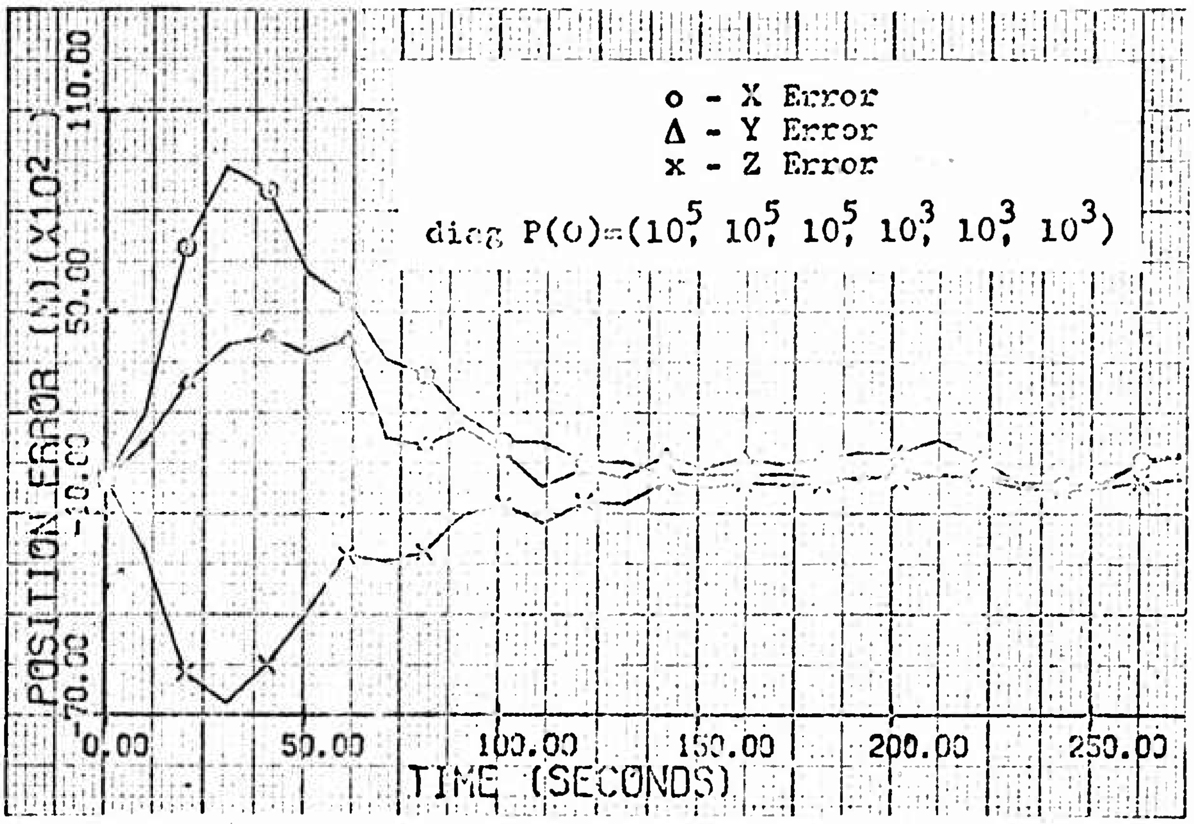

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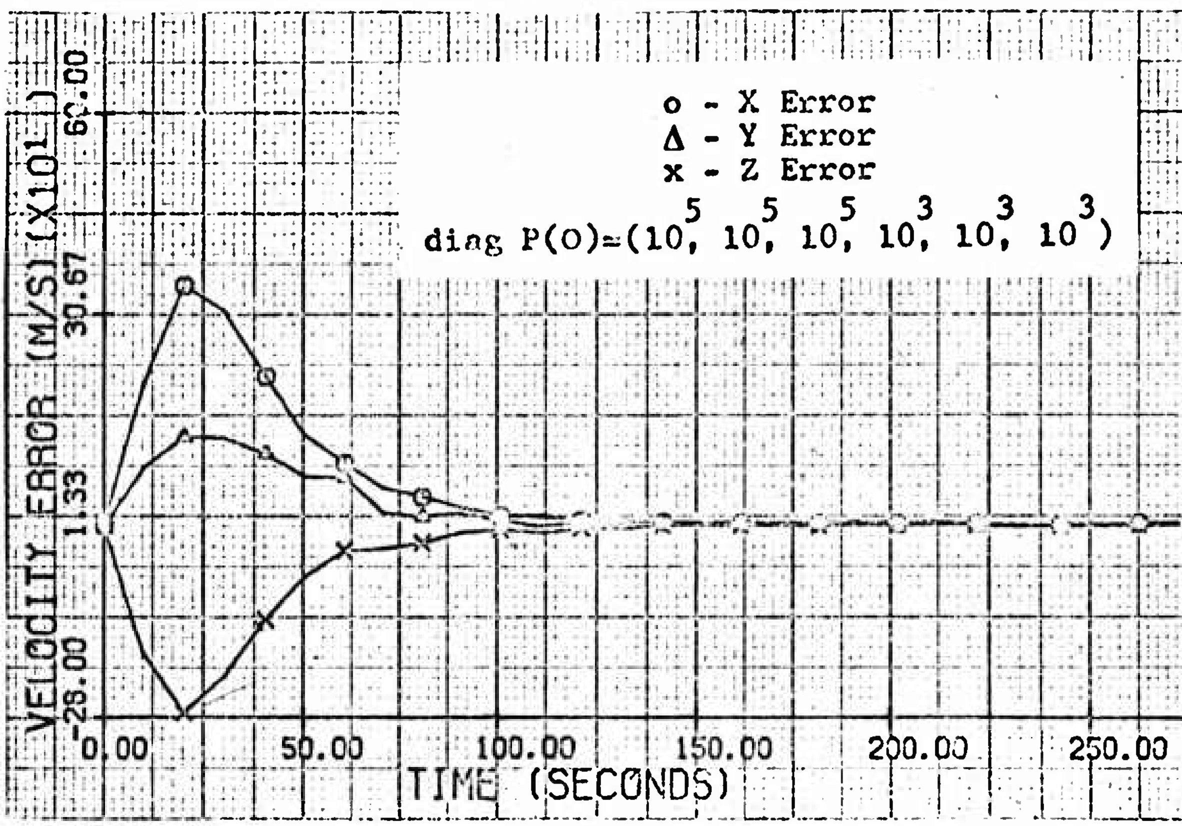

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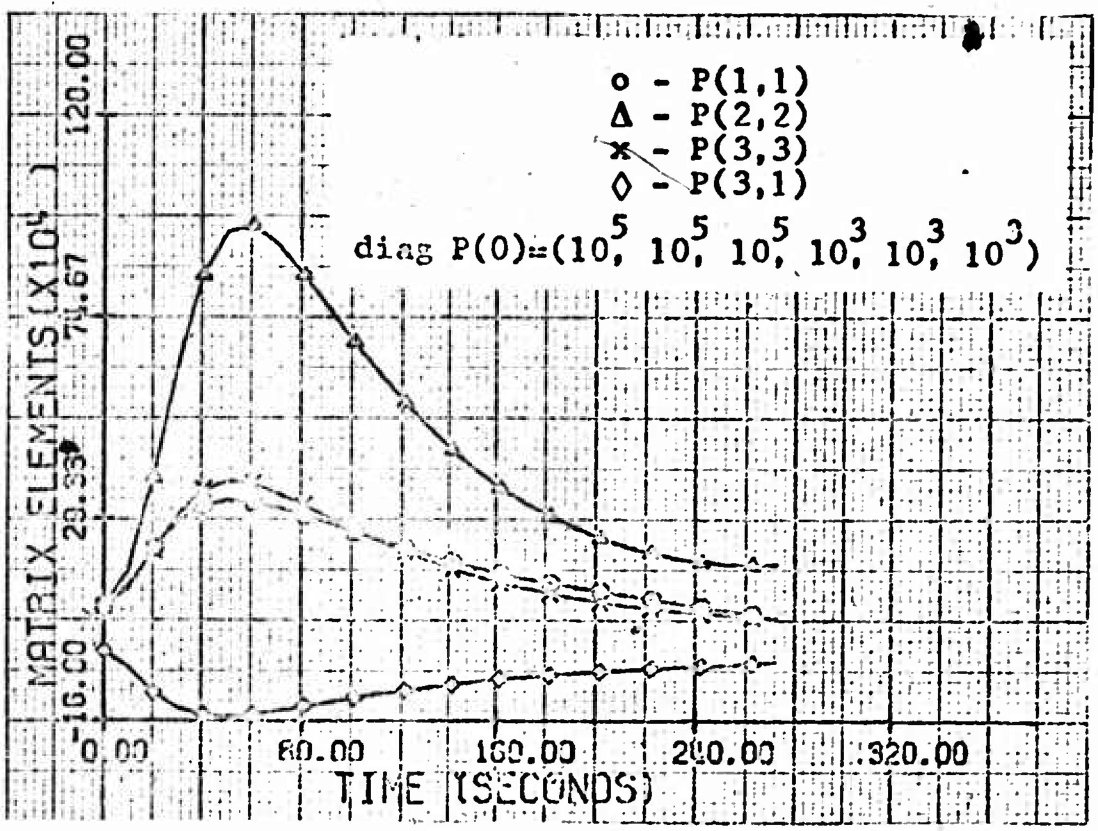

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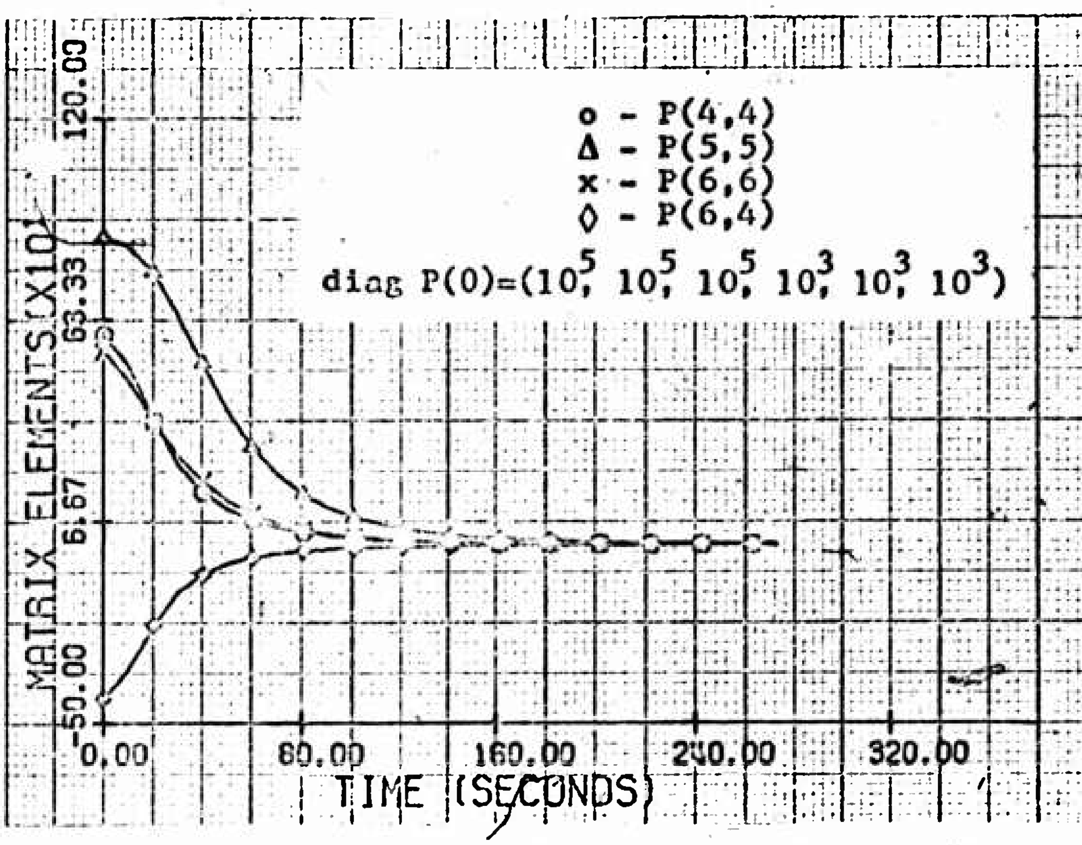

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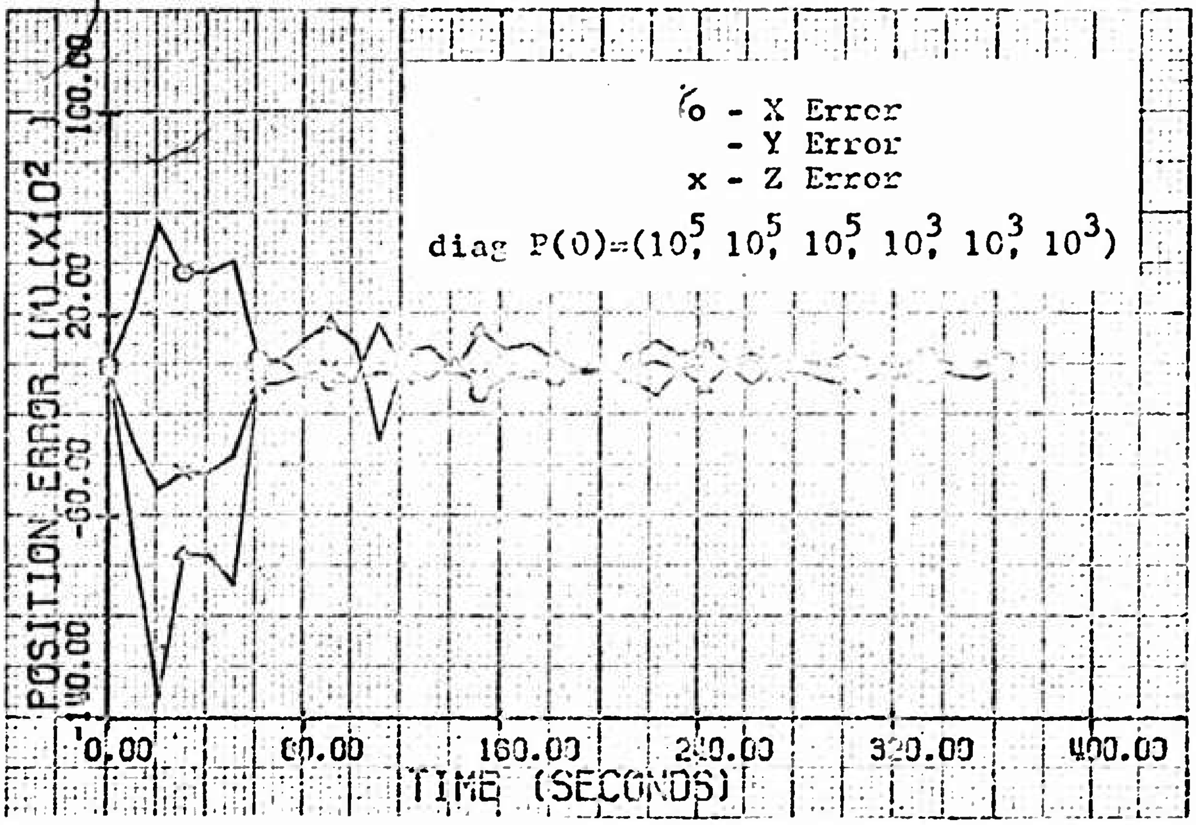

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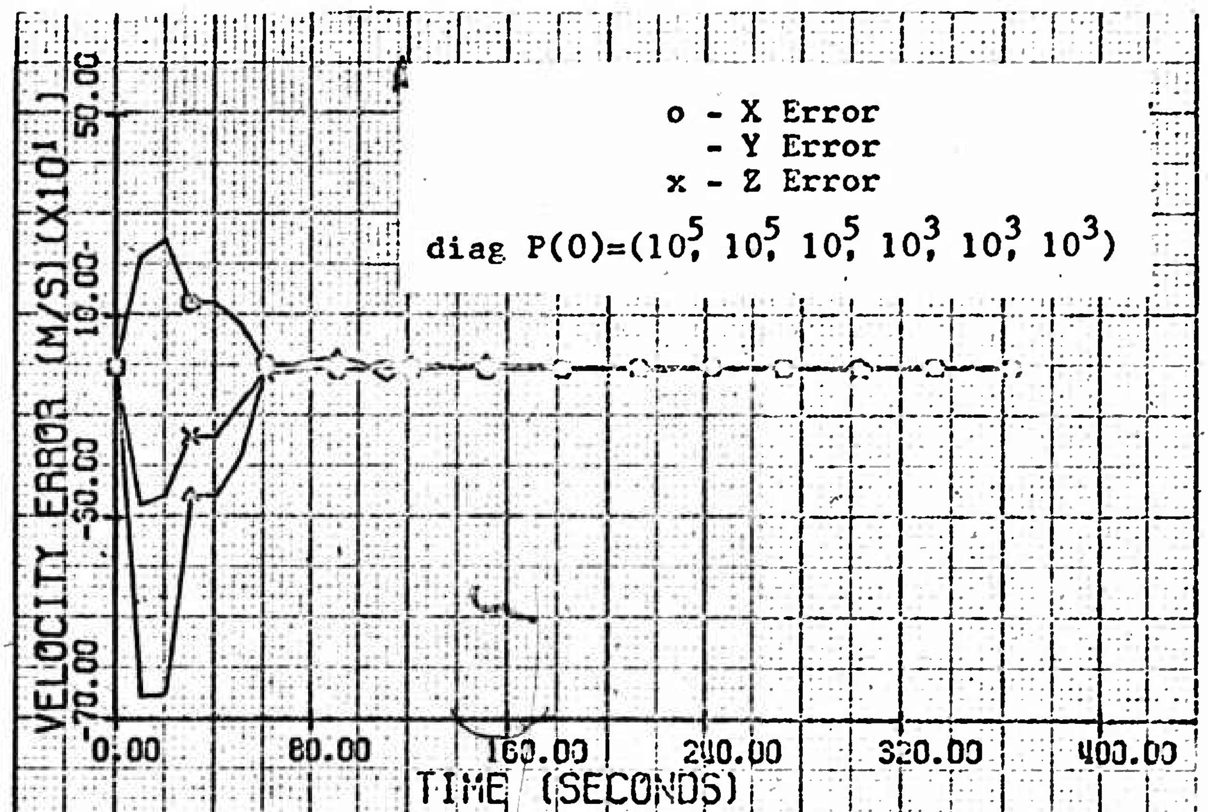

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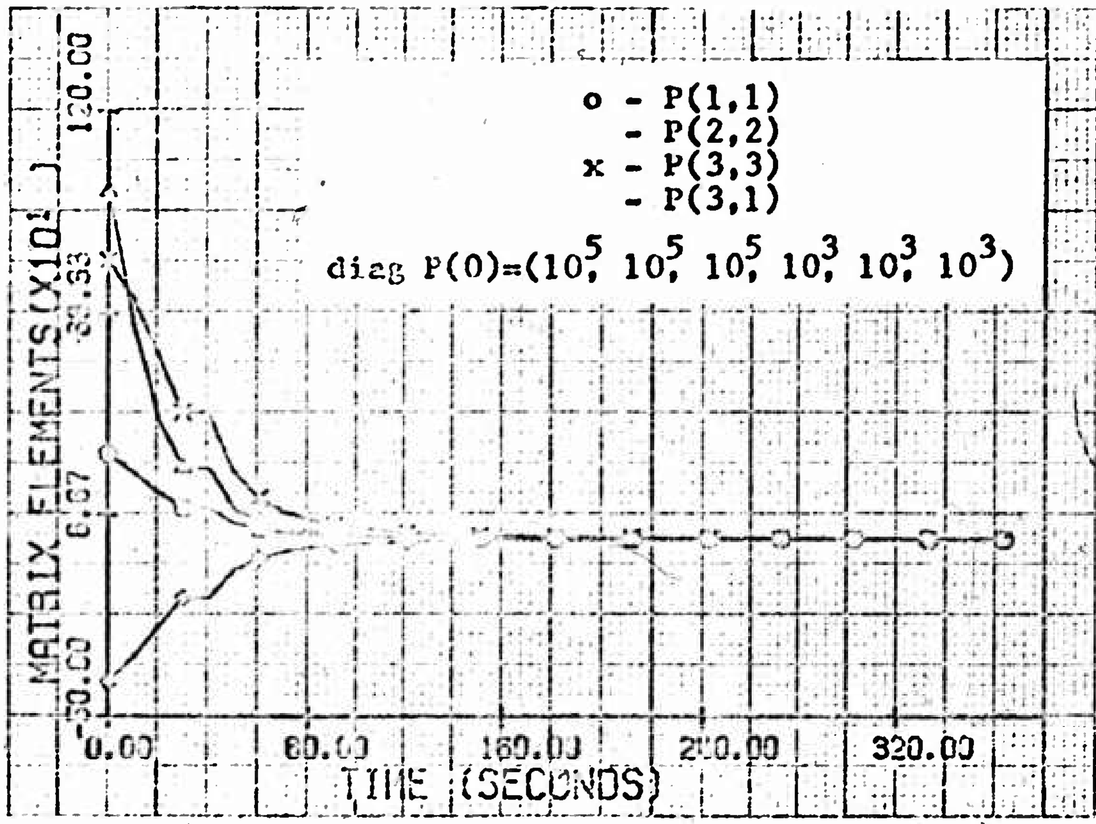

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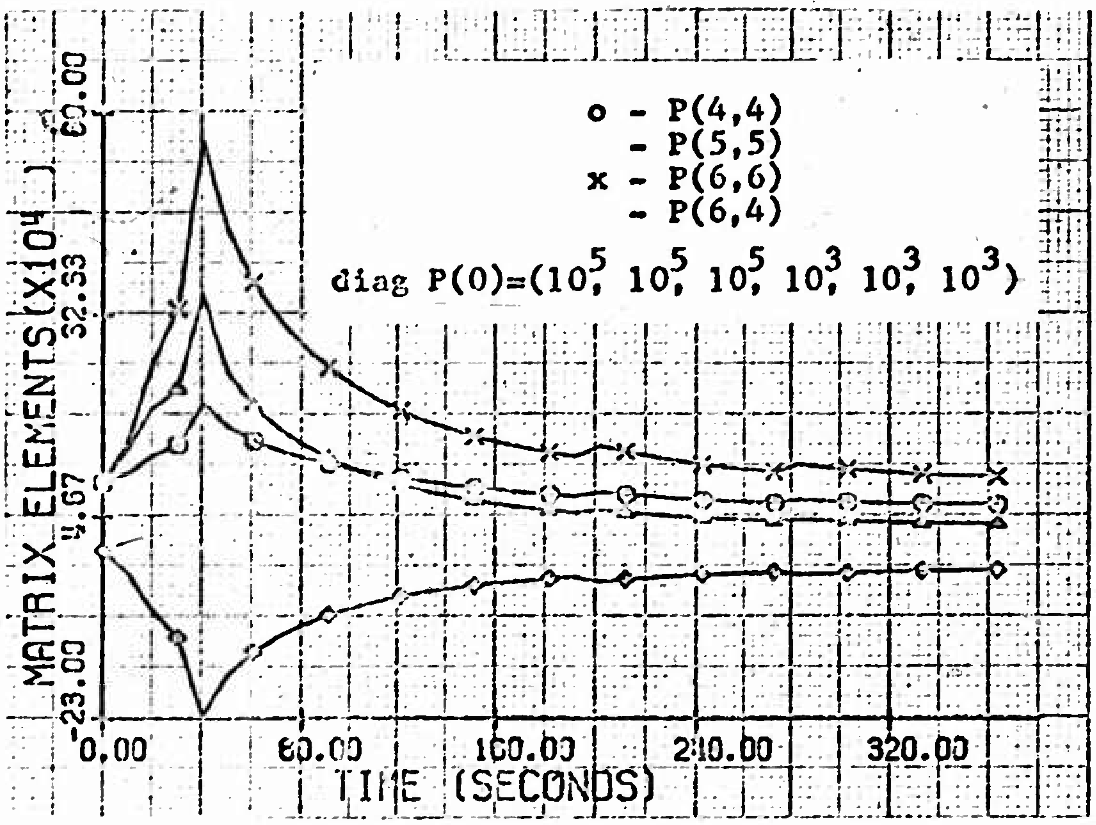

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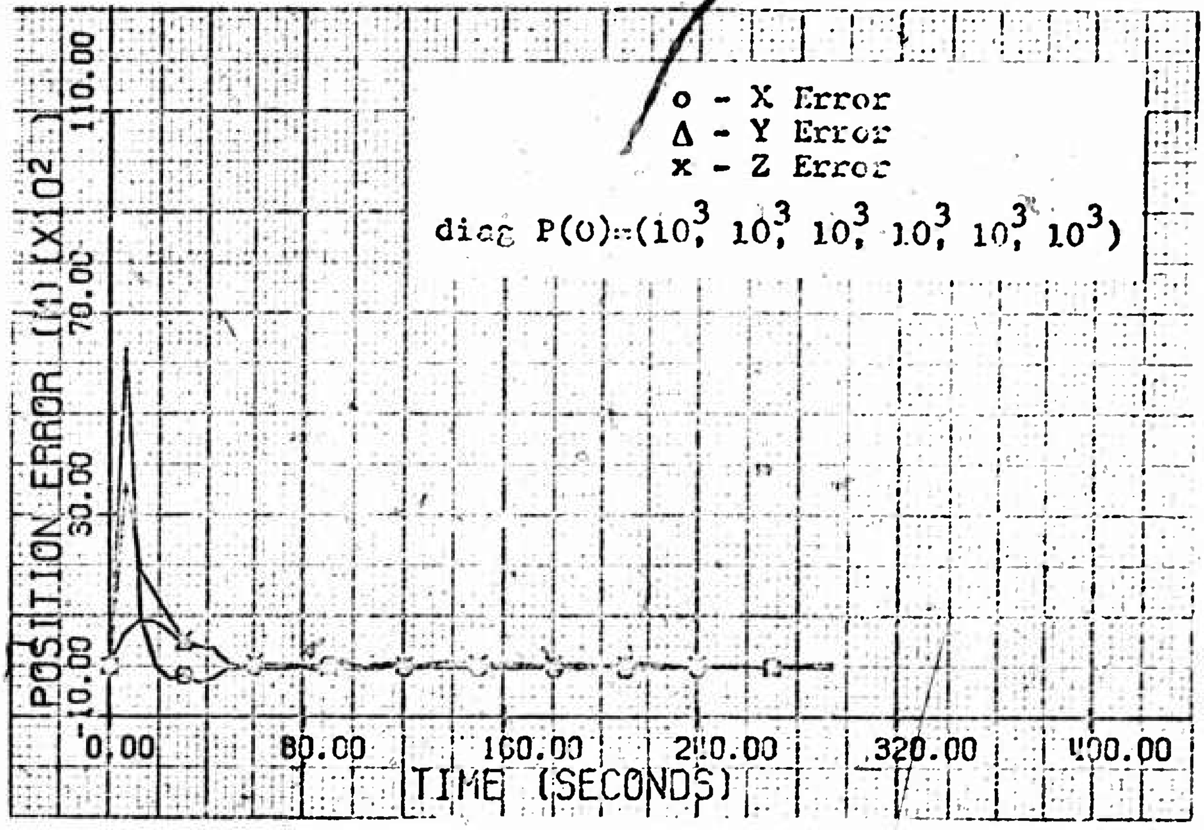

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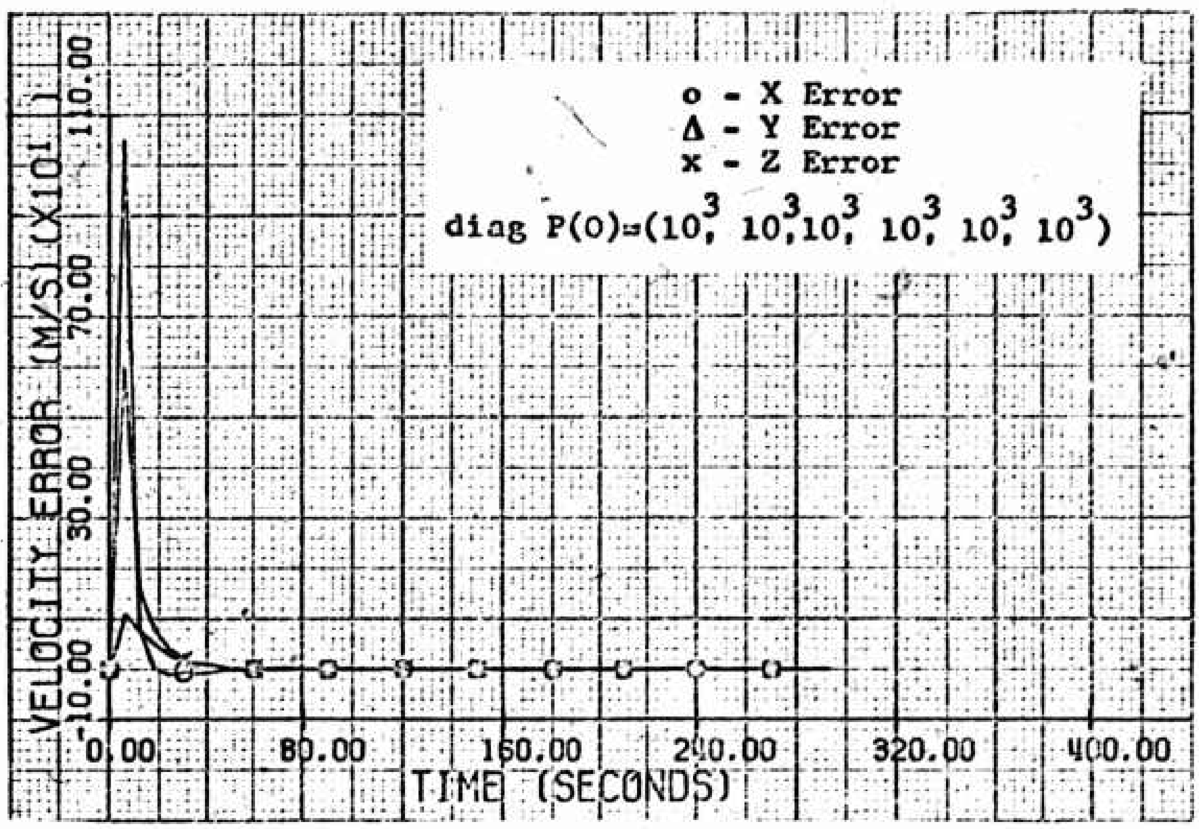

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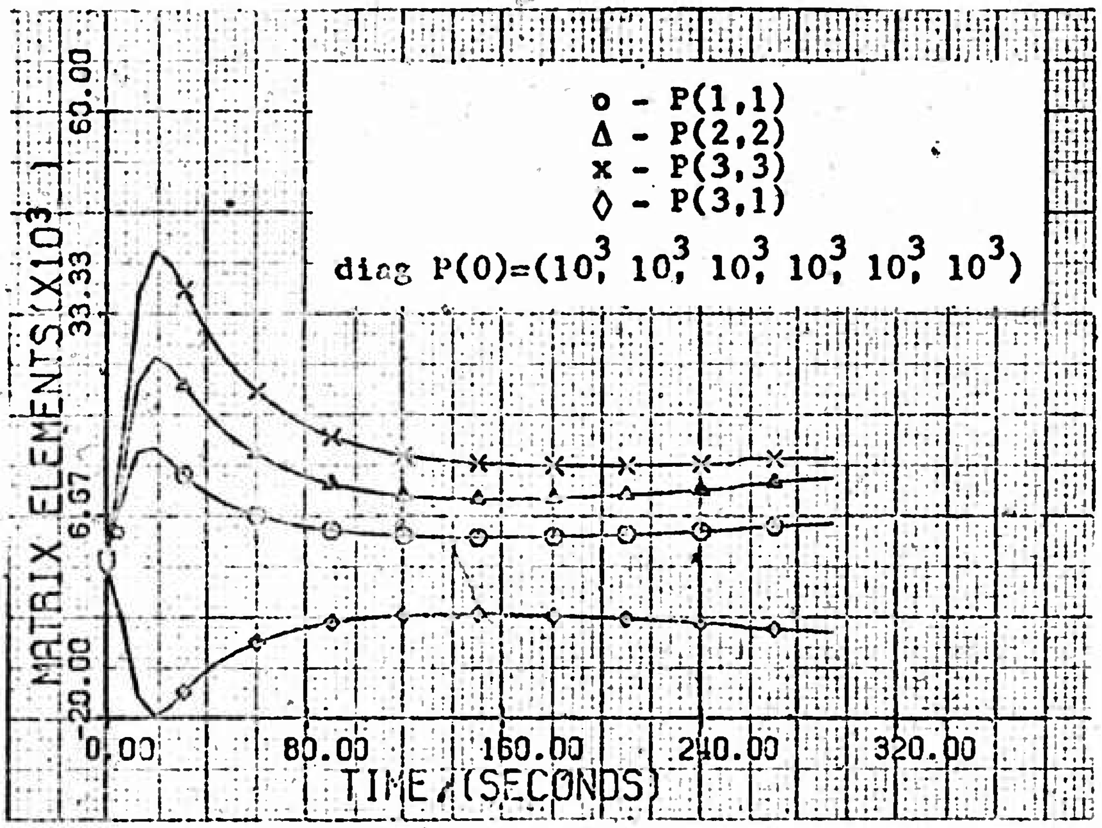

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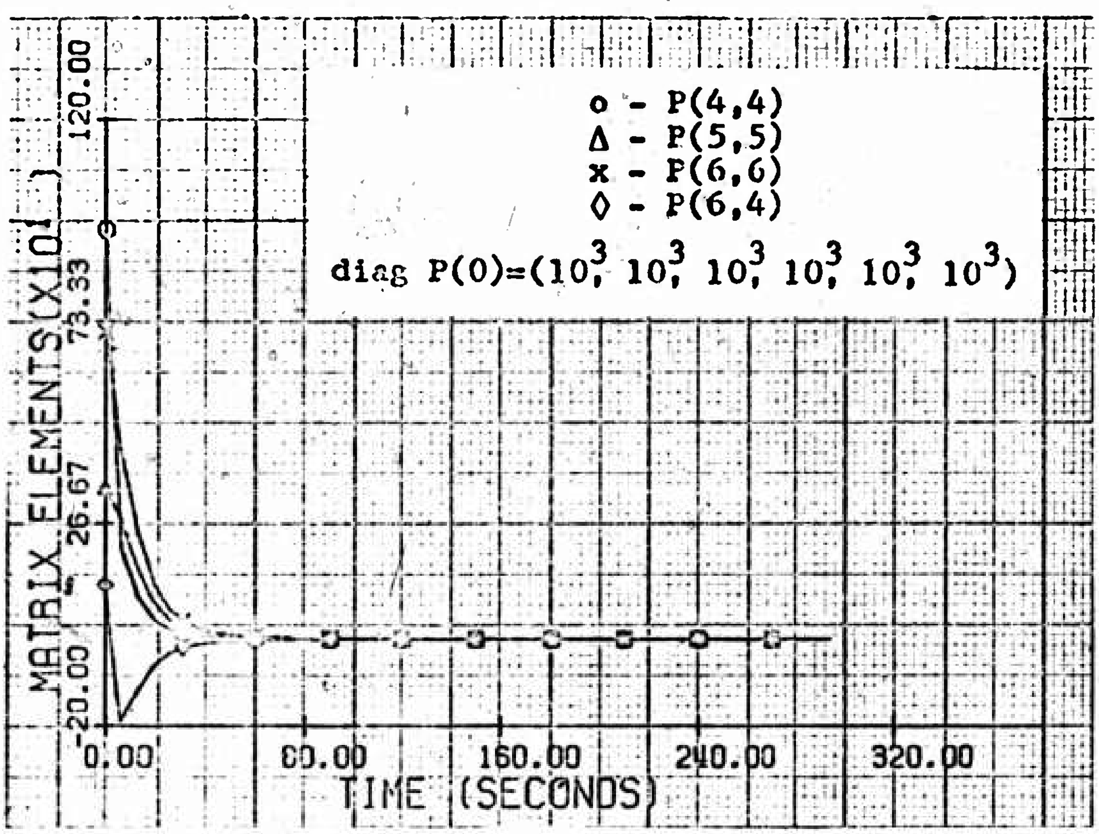

Fig. 98 Satellite 3823 Station 345 Pass #9

VITA: Thomas R. Filiatreau

Thomas R. Filiatreau was born [redacted] He was graduated from [redacted] June, 1954. He enlisted in the USAF on 23 December 1954 and served as a medical laboratory technician in several USAF hospitals until August 1961. He then enrolled at the University of Oklahoma under the AFIT Airman's Education and Commissioning Program. He was graduated from University of Oklahoma with a Bachelor of Science in Electrical Engineering and was commissioned in the USAF on 28 September 1964. He was an electronic engineer assigned to the 341 Strategic Missile Wing (SAC), Malmstrom AFB, Montana prior to entering the AFIT Resident School at Wright-Patterson AFB, Ohio in August 1968.

VITA: George Edward Elliott

George Edward Elliott was born [redacted] He completed his studies at [redacted] 1959, and enlisted in the USAF in October 19, 1959. After attending several in-service electronics schools, he instructed in the Bomber and Navigation Trainer School, the Skybolt Missile program, and the F4C Fire Controls School, all located at Lowry AFB, Colorado. He was an F4C radar technician on Okinawa from November 1964 until September 1965 when he enrolled at Colorado State University under the AFIT Airman's Education and Commissioning Program. Upon graduation with a Bachelor of Science degree in Electrical Engineering in March 1968, he was commissioned in the USAF on 27 June 1968. He entered the AFIT Resident School at Wright-Patterson AFB in August 1968, and is a member of Sigma Tau, Eta Kappa Nu, and Tau Beta Pi.

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13. ABSTRACT

As more satellites are launched into space, a great need exists for a rapid, recursive (reducing computer memory requirements) orbit determination method. This paper presents such a method by applying Kalman Filter theory to determine the position and velocity of near-earth satellites using data from a fixed observer on the Earth. The vehicle equations of motion are linearized in a Taylor Series expansion The nominal states, position and velocity, are determined by integrating the nonlinear equations of motion, and the linear filter theory is used to estimate the errors in these states. The linear estimated errors are added to the nominal states to obtain an updated trajectory which is used as the starting point on a new nominal for the next integration. Actual tracking data from four different satellites are used in the study. Convergence of the error estimates to values less than 0.1 per cent of the best estimates of position and velocity is obtained within 50-250 seconds from the time of the initial radar contact. The program is capable of integrating for over 80 seconds with no tendency to diverge. Several orbital elements are computed, and these compare quite closely with results supplied by the Space Detection and Tracking System. The rate of convergence is related to the initial guess of the error covariance matrix, along with the measurement accuracy of the tracking stations. Results are presented in both tabular and graphical form.

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